Facility location I.

Chapter 10

Facility location

Continuous facility location models

- Single facility minisum location problem
- Single facility minimax location problem

Facility location

- Factors that influence the facility location decision:
  - Transportation (availability, cost)
  - Labor (availability, cost, skills)
  - Materials (availability, cost, quality)
  - Equipment (availability, cost)
  - Land (availability, suitability, cost)
  - Market (size, potential needs)
  - Energy (availability, cost)
  - Water (availability, quality, cost)
  - Waste (disposal, treatment)
  - Financial institutions (availability, strength)
  - Government (stability, taxes, import and export restrictions)
  - Existing plants (proximity)
  - Competitors (size, strength and attitude in that region)
  - Geographical and weather conditions

Facility location

- Facility location problem
  - Site pre-selection (qualitative)
  - Pre-selected sites evaluation (quantitative)
    - Factor Rating
    - Cost-Profit-Volume analysis
- Continuous facility location problem
  - Facility location models
  - Choice of ANY site in the space
  - The sole consideration is transportation cost

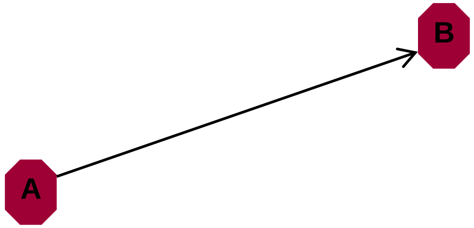
Continuous facility location problems

- For the new facility we can choose ANY site in the space
- For the existing related facilities (suppliers, customers, *etc.*) we know the coordinates (*x,y*) and the flows (cost) between them and the new facility
- The sole consideration is **transportation cost**
- Facility location models have numerous applications
  - New airport, new hospital, new school
  - Addition of a new workstation
  - Warehouse location
  - Bathroom location in a facility *etc.*

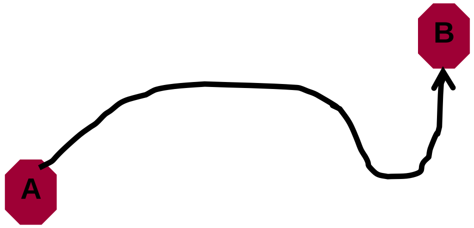
Continuous facility location problems
Distance measures

- **Rectilinear distance**
  - Along paths that are orthogonal or perpendicular to each other
  - $|x_1 - x_2| + |y_1 - y_2|$

- **Euclidean distance**
  - Straight line between two points
  - $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$



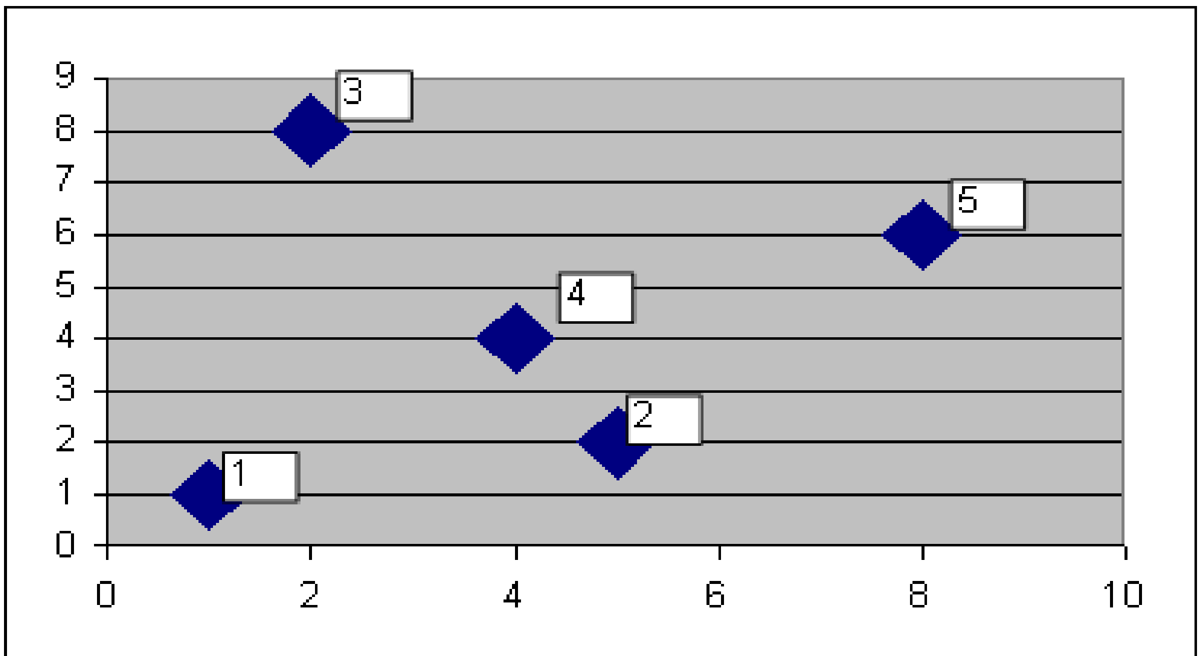
- **Flow path distance**
  - Exact traveling distance between two points



Rectilinear Facility Location Problems

Example

- Determine a new location of a warehouse in Montreal area which provides materials to 5 different companies
- Location of these companies (a, b) and the material movement between the new warehouse and the existing facilities (w) are provided:
- Where should the new warehouse be located?



a	b	w
1	1	5
5	2	6
2	8	2
4	4	4
8	6	8

Rectilinear Facility Location Problems

- Various objectives can be used
  - **Minisum location problem**
    - Minimizing the sum of weighted distance between the new facility and the other existing facilities
  - **Minimax location problem**
    - Minimizing the maximum distance between the new facility and any existing facility

Single-facility minisum location problem

Objective function: $$\min\ f(x) = \sum_{i=1}^{m} w_i d(X, P_i)$$

Distances in rectilinear models: $$d(X, P_i) = |x - a_i| + |y - b_i|$$

- Where
  - $X = (x, y)$ Location of new facility
  - $P = (a_i, b_i)$ Locations of existing facilities
  - w_i "weight" associated with travel between the new facility and existing facility *i*
  - $d(X, P_i)$ distance between the new facility and existing facility *i*

Single-facility minisum location problem

- Find the ***x*** and ***y*** values for the new facility that satisfy the given objective

$$\min\ f(x) = \sum_{i=1}^{m} \mathrm{w}_{\mathrm{i}} \left| x - a_i \right| + \sum_{i=1}^{m} w_i \left| y - b_i \right|$$

- Apply these rules to find the optimum value of *x*:
  1. *X-coordinate* of the new facility will be the same as the *x*-coordinate of some existing facility
  2. *Median condition:* Selected *X* coordinate cannot be more than half the total weight which is to the right of *x*, or which is to the left of *x*.
- Same rules apply in selection of the optimum value of y

Single-facility minisum location problem

- Procedure

1. Find ***x*-coordinate**:
   - Order the facilities based on the ascending order of their ***x-coordinates***
   - Calculate partial sum of weights
   - Find the facility for which the partial sum first equals or exceeds one-half the total weight
   - The ***x-coordinate*** of the new facility will be the same as the ***x-coordinate*** of this facility
2. Find ***y*-coordinate**
   - Repeat the same for ***y-coordinate***

Single-facility minisum location problem
Alternate sites

- If we cannot place the new facility on the selected location, then alternate sites could be evaluated by computing the *f(x,y)* values for all the possible locations and chose the location that gives the minimum *f(x,y)* value.

$$\min\ f(x) = \sum_{i=1}^{m} \mathrm{w}_{\mathrm{i}} |x - a_i| + \sum_{i=1}^{m} w_i |y - b_i|$$

Single-facility minisum location problem
Example

- A new location for a manufacturing facility is being considered. The facility has frequent relationships with its five major suppliers and since the supplied material is bulky and transportation costs are high the closeness to the five suppliers has been determined as the major factor for the facility location. The current coordinates of the suppliers are S1=(1,1), S2=(5,2), S3=(2,8), S4=(4,4) and S5=(8,6). The cost per unit distance traveled is the same for each supplier, but the number of trips per day between the facility and each of its suppliers are 5,6,2,4 and 8.

- ➢ Find a new location for the facility which minimizes the transportation costs
- ➢ Calculate total weighted distance for the new location.
- ➢ If the facility cannot be placed in the optimal location, find the second best alternative site out of (5,6), (4,2) and (8,4) .

Single-facility minisum location problem

Example

Supplier i	Coordinates a_i	Coordinates b_i	Relationship with the facility (trips per day) w_i
1	1	1	5
2	5	2	6
3	2	8	2
4	4	4	4
5	8	6	8

1. Rank the X coordinates

Supplier i	Coordinates a_i	Relationship with the facility (trips per day) w_i	$\sum_{i=1}^{5} w_i$
1	1	5	5
3	2	2	7
4	4	4	$11<25/2$
2	**5**	**6**	**$17>25/2$**
5	8	8	25

Find *x*-coordinate:

- Order the suppliers based on the ascending order of their *x*-coordinates
- Calculate partial sum of weights
- Find the supplier for which the partial sum first equals or exceeds one-half the total weight
- The *x*-coordinate of the new facility will be the same as the one of this supplier

Half the total weight:
(5+2+4+6+8)/2 = 25/2 =12.5

Rule 1: here the partial sum first equals or exceeds ½ the total weight

Rule 2: Selected X should be the same as the x-coordinate of some of the suppliers (here S2)

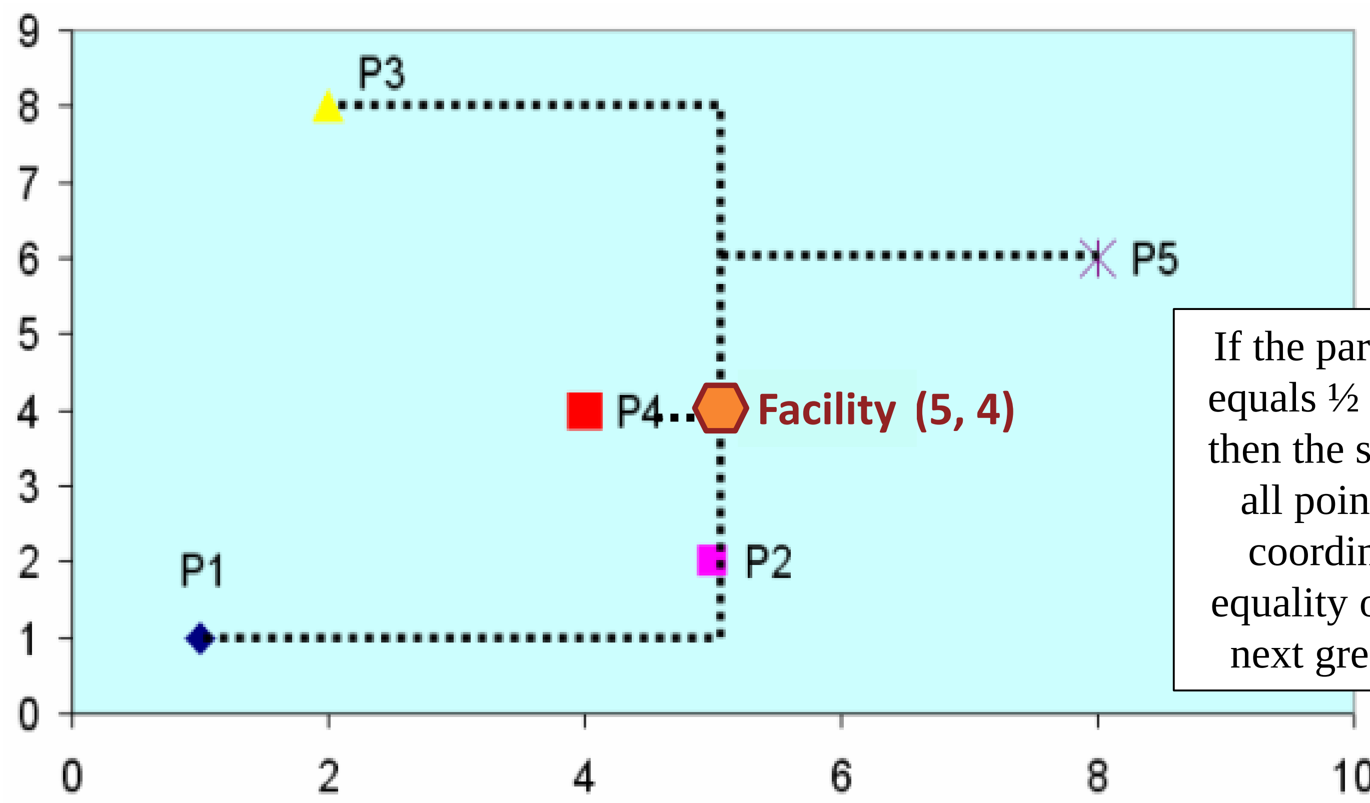
2. Rank by the Y coordinates

Supplier i	Coordinates	Relationship with the facility (trips per day)	
	b_i	w_i	$\sum_{i=1}^{5} w_i$
1	1	5	5
2	2	6	11<25/2
4	4	4	15>25/2
5	6	8	23
3	8	2	25

Repeat for y-coordinate:

Rule 1: here the partial sum first equals or exceeds ½ the total weight

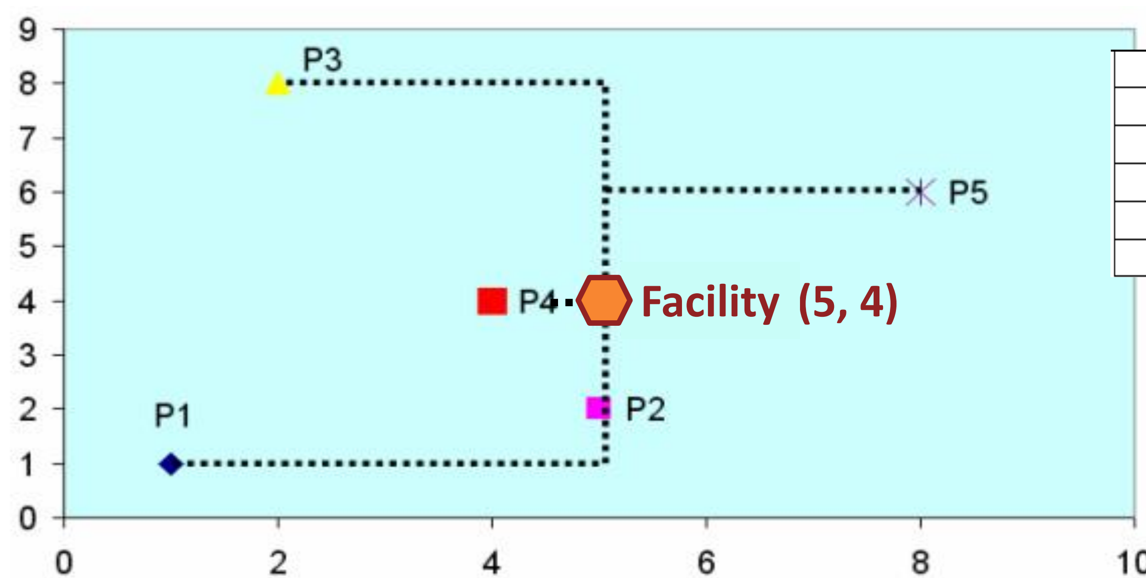
Rule 2: Selected Y should be the same as the **y-coordinate** of some of the suppliers (here S4)



If the partial sum exactly equals ½ the total weight, then the solution includes all points between the coordinate where the equality occurred and the next greater coordinate

- The best location for the new facility corresponds to the coordinates x = 5 and y = 4
- The total weighted distance between the new facility and its suppliers can be found as:

$$f(x) = \sum_{i=1}^{m} w_i |x - a_i| + \sum_{i=1}^{m} w_i |y - b_i|$$



i	a_i	b_i	w_i
1	1	1	5
2	5	2	6
3	2	8	2
4	4	4	4
5	8	6	8

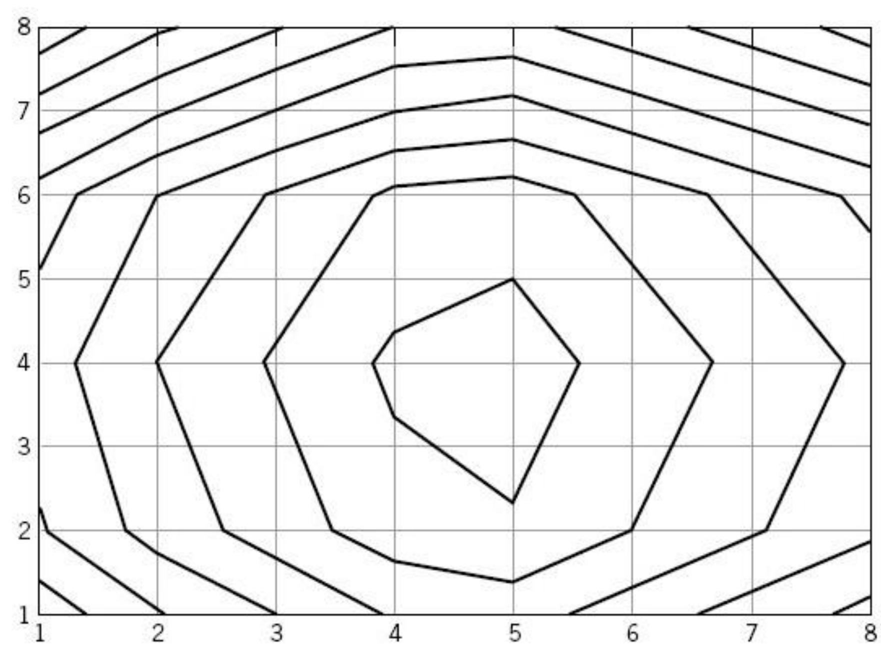
$$f(5,4) = \left\{ \begin{matrix} 5(|5-1| + |4-1|) + 6(|5-5| + |4-2|) + 2(|5-2| + |4-8)| \\ +4(|5-4| + |4-4) + |8(|5-8| + |4-6|) \end{matrix} \right\} = 105$$

Single-facility minisum location problem
Example

- $O_5 \rightarrow f(8,4) = 50+30+20+16+16 = 132$
- $O_6 \rightarrow f(5,6) = 45+24+10+12+24 = 115$
- $O_7 \rightarrow f(4,2) = 20+6+16+8+64 = 114$
- If these are the only options available, then we would select the location 7 to place the new facility

Single-facility minisum location problem
Iso-cost contour lines

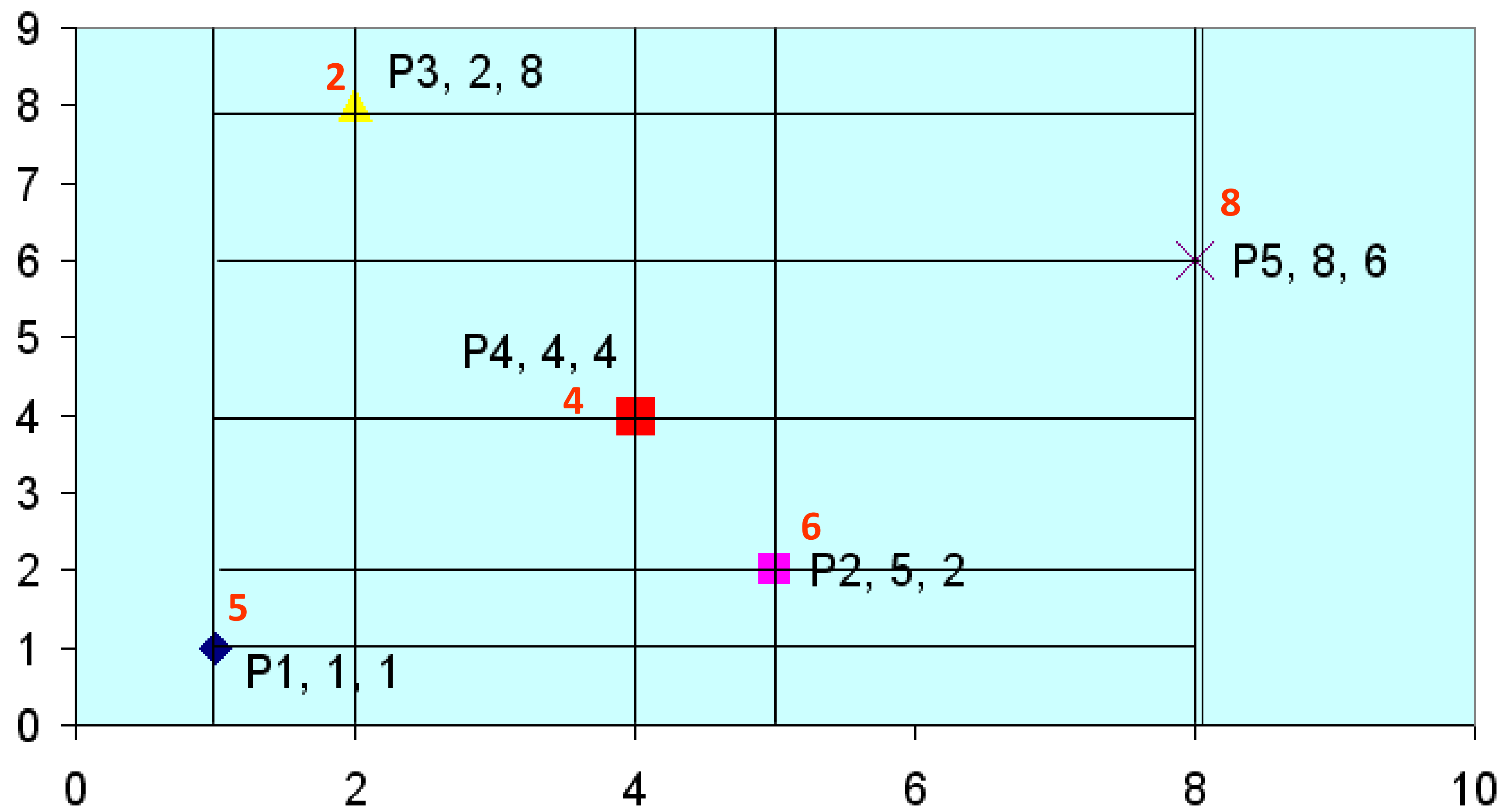
- Iso-cost contour lines
  - Designate movement that does not change the value of the objective function
  - Can help in determining an appropriate location for a new facility.



Single-facility minisum location problem
Iso-cost contour lines

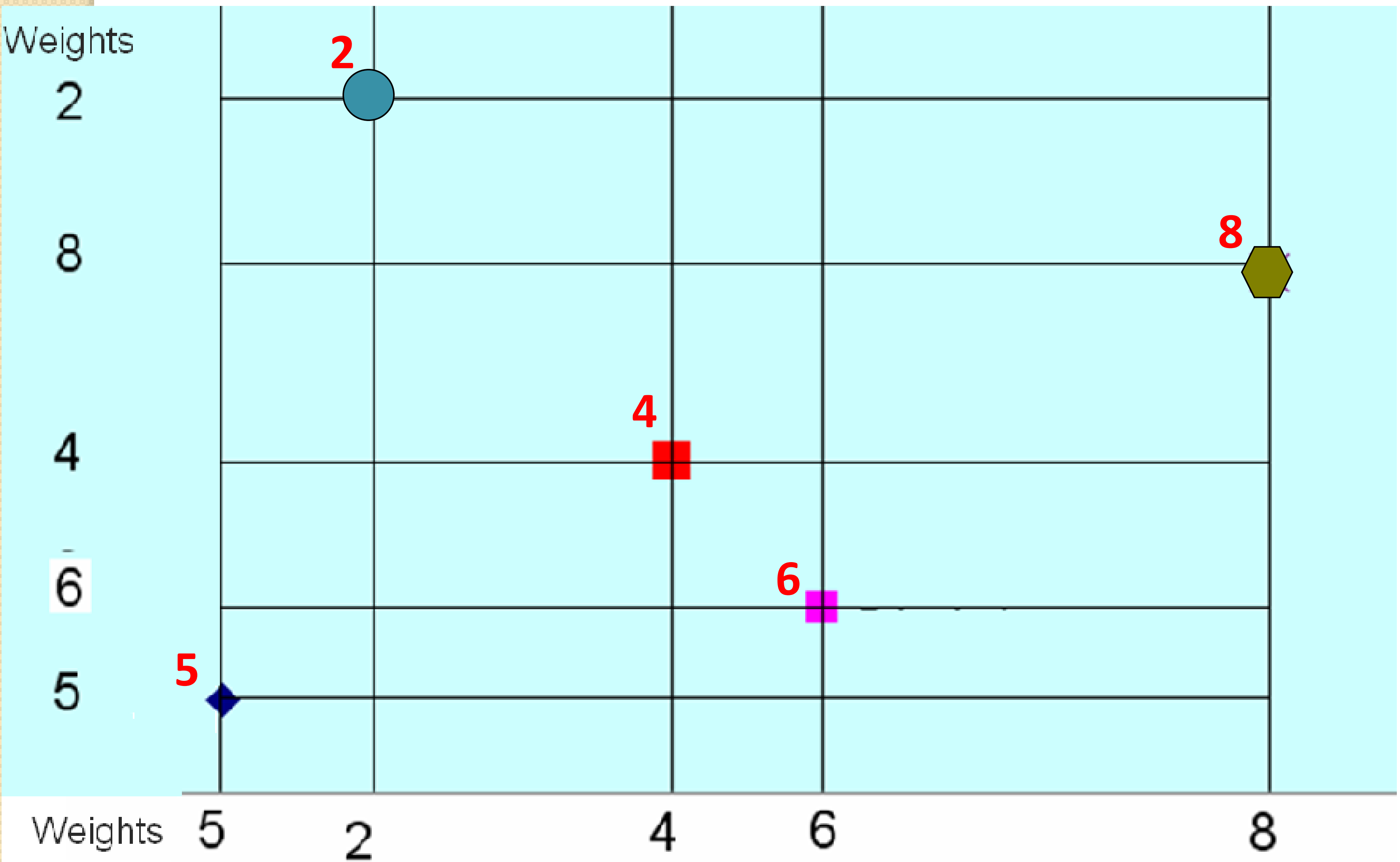
- Procedure:
  1. Plot the locations of existing facilities
  2. Draw vertical and horizontal lines through each existing facility
  3. Sum the weights for all existing facilities having the same x-coordinate and enter the total at the bottom of the vertical lines. Do the same for y coordinates
  4. Calculate “net pull” for each candidate x- coordinate. (pull to the right is positive and pull to the left is negative). Do the same for y coordinates
  5. Determine the slope for each grid region enclosed by the candidate coordinates
     - The slope equals the negative of the ratio of the net horizontal pull and the net vertical pull
  6. Construct an iso-cost contour line from any candidate coordinate point by following the appropriate slope in each grid.

1. Plot the locations of existing facilities
2. Draw vertical and horizontal lines through each existing facility

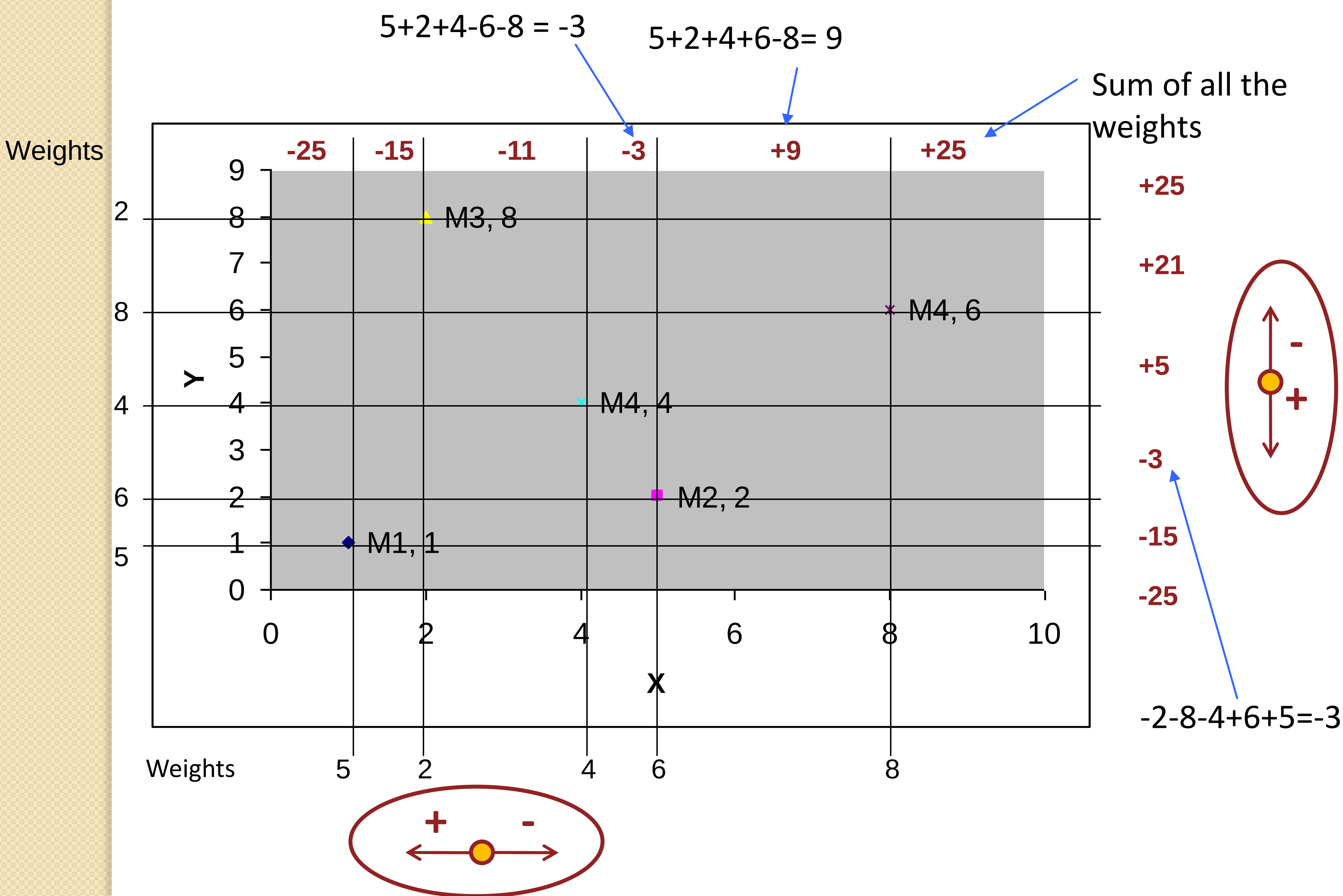


- Weights are given **in RED,** and the coordinates of the existing facilities are given **in BLACK.**
- According to the minisum algorithm **the intersections of the lines are considered as the candidate locations** of the new facility.

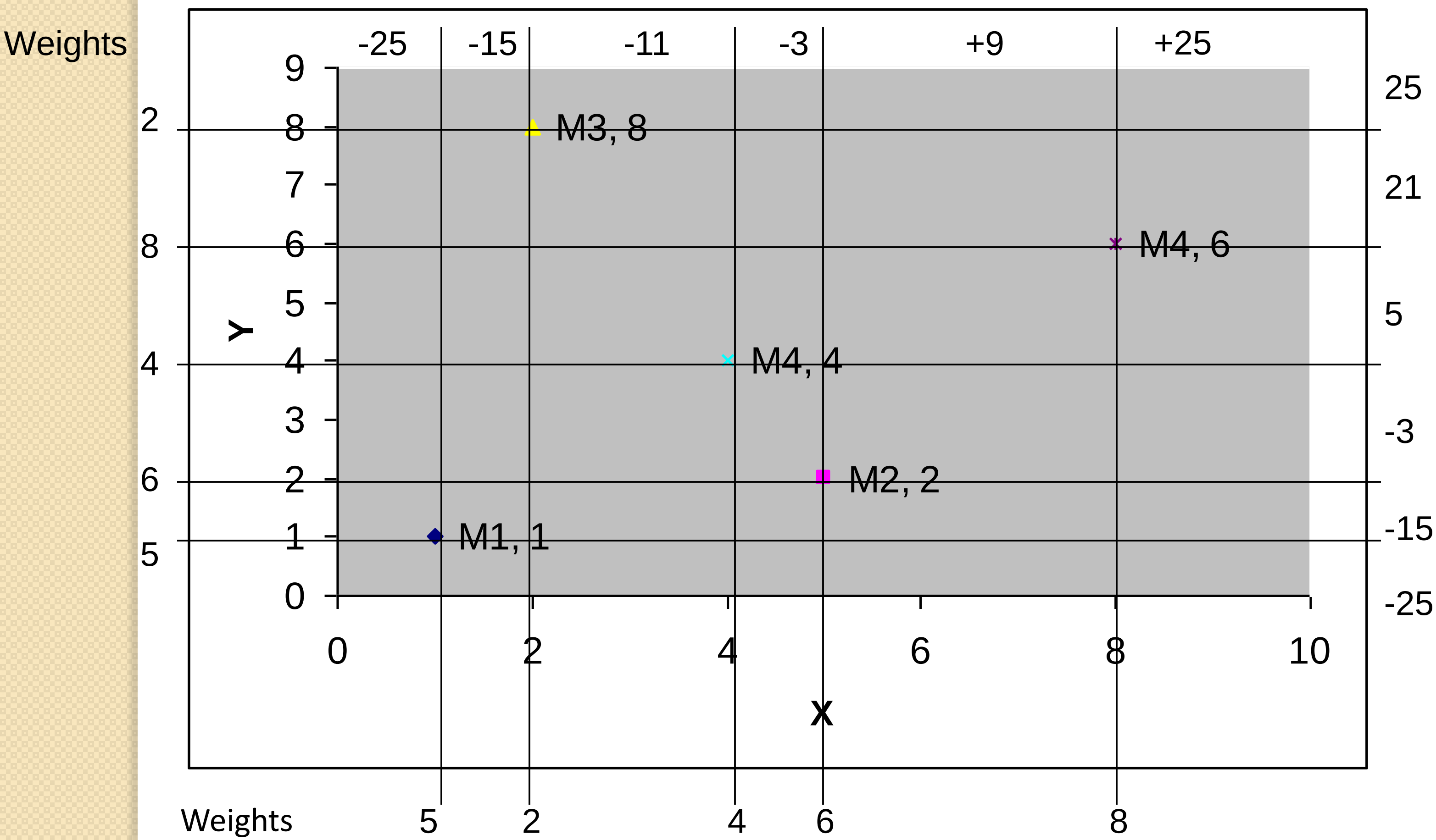
3. **Weights** are placed on *x* and *y* coordinates



4. Calculate "***net pull***" for each candidate x- coordinate. Pull to the right is negative and pull to the left is positive. Do the same for y coordinates, where pull up is negative and pull down is positive.



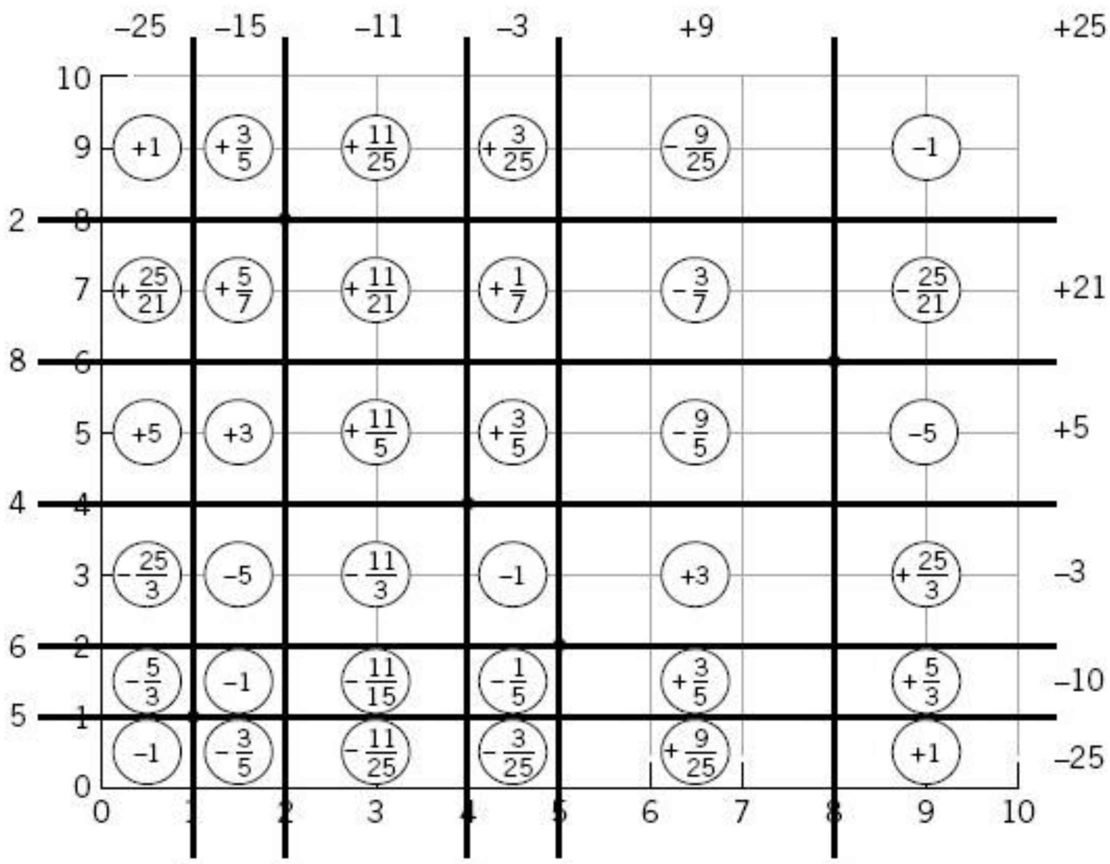
Net horizontal and vertical “forces” for regions defined by candidate coordinates



Single-facility minisum location problem

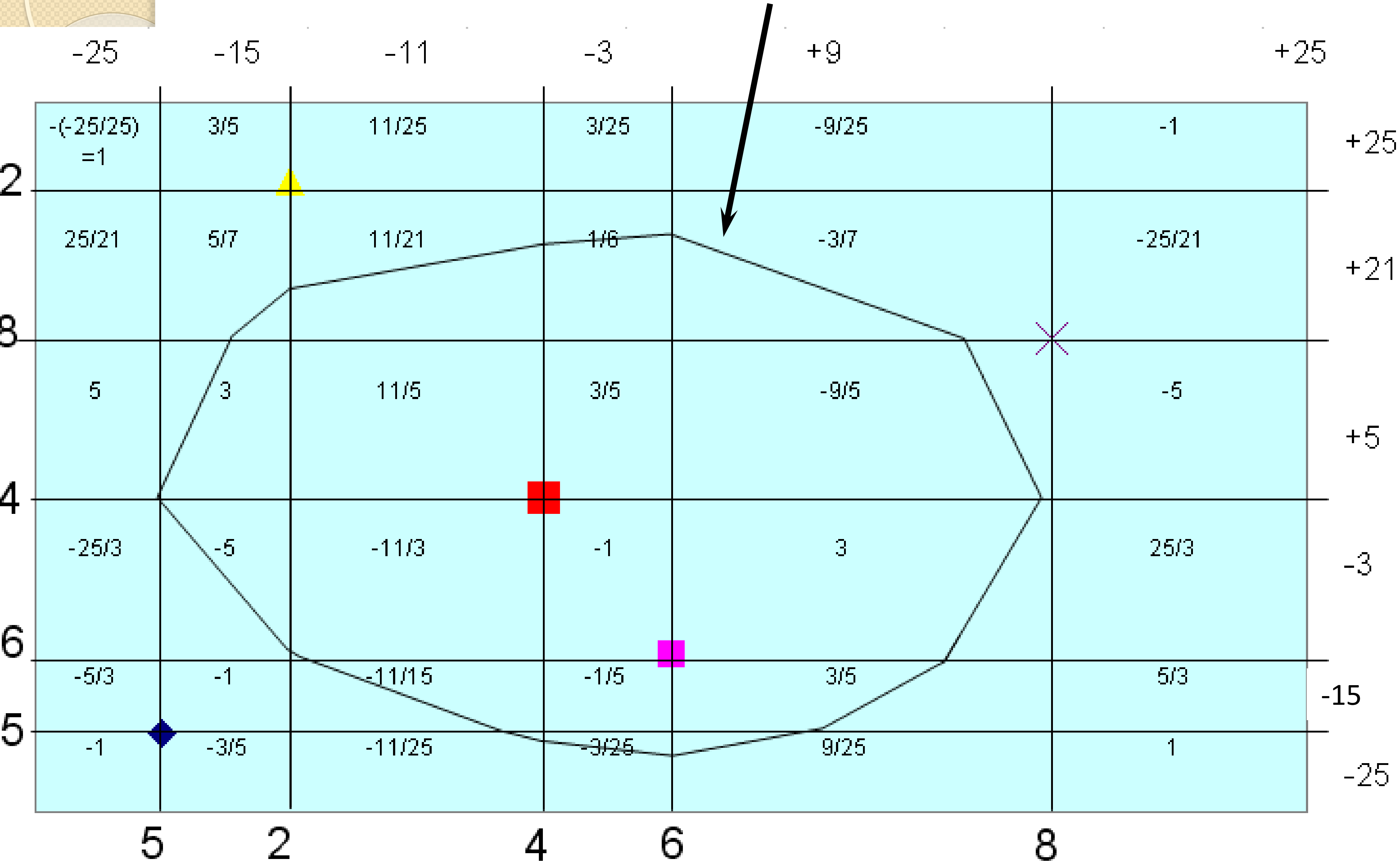
Iso-cost contour lines

5. Determine the slope for each grid region enclosed by the candidate coordinates. The slope equals the negative of the ratio of the net horizontal pull and the net vertical pull



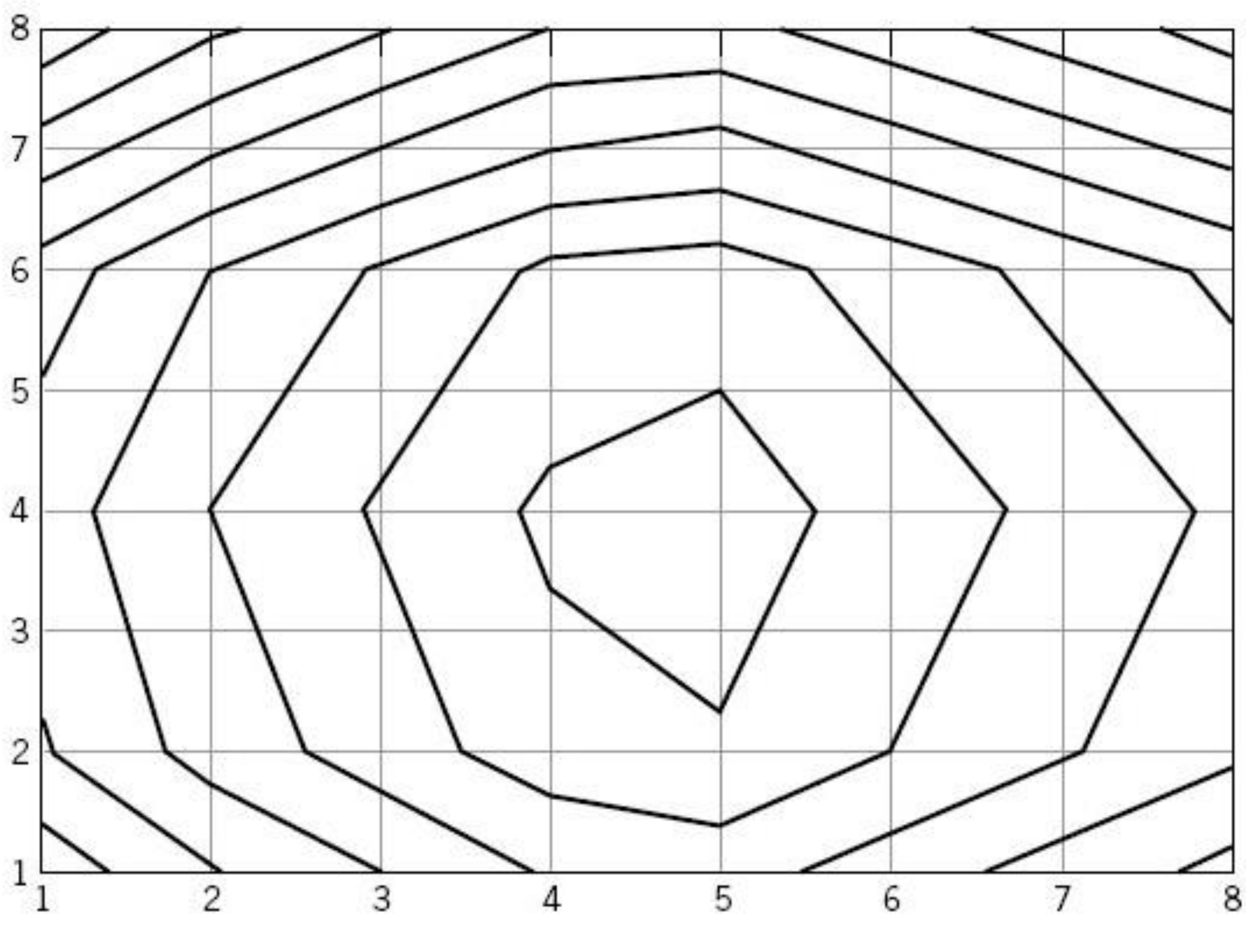
$$-\frac{\text{Horizontal pull}}{\text{Vertical pull}}$$

6. Construct an iso-cost contour line from any candidate coordinate point by following the appropriate slope in each grid.



Single-facility minisum location problem
Iso-cost contour lines

Sample iso-contour lines:



Single facility minimax location problem

- The objective is to minimize the maximum distance between the new facility and any existing facility
  - The objective function:

    Minimize $f(X) = \max[(|x - a_i| + |y - b_i|), i = \{1, 2, .. M\}]$
- Procedure:
  - To obtain a minimax solution, let
    - c_1 = minimum $(a_i + b_i)$
    - c_2 = maximum $(a_i + b_i)$
    - c_3 = minimum $(-a_i + b_i)$
    - c_4 = maximum $(-a_i + b_i)$
    - $c_5 = \max(c_2 - c_1, c_4 - c_3)$
  - Optimum solution for the new facility location is on the line segment connecting the points $X_1^*(x_1^*, y_1^*)$ and $Y_2^*(x_2^*, y_2^*)$
    - $X_1^*(x_1^*, y_1^*) = 0.5(c_1 - c_3, c_1 + c_3 + c_5)$
    - $Y_2^*(x_2^*, y_2^*) = 0.5(c_2 - c_4, c_2 + c_4 - c_5)$
    - Max distance equals $c_5/2$

Single facility minimax location problem

Example

- A company which has already eight facilities intends to build another one and is currently looking for the most convenient location. It was determined that the most appropriate place is the one which is closest to the existing facilities. The locations of the current facilities are given below. Find the best minimax locations for an additional facility. What will be the maximum distance to any other facility?

i	a_i	b_i
1	0	0
2	4	6
3	8	2
4	10	4
5	4	8
6	2	4
7	6	4
8	8	8

Single facility minimax location problem

Example

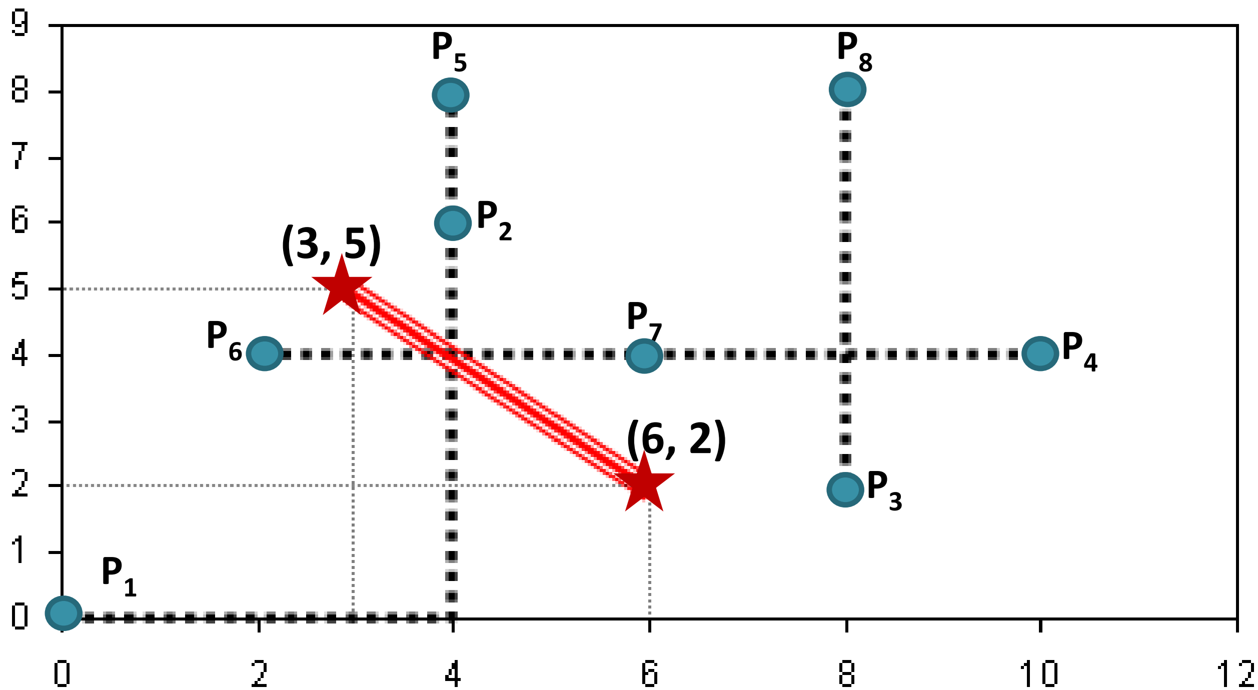
i	*a*	*b*	$a_i + b_i$	$-a_i + b_i$
1	0	0	**0**	**0**
2	4	6	**10**	**2**
3	8	2	**10**	**-6**
4	10	4	**14**	**-6**
5	4	8	**12**	**4**
6	2	4	**6**	**2**
7	6	4	**10**	**-2**
8	8	8	**16**	**0**

- c_1 = minimum $(a_i + b_i)$
  $\mathbf{c_1 = 0}$
- c_2 = maximum $(a_i + b_i)$
  $\mathbf{c_2 = 16}$
- c_3 = minimum $(-a_i + b_i)$
  $\mathbf{c_3 = -6}$
- c_4 = maximum $(-a_i + b_i)$
  $\mathbf{c_4 = 4}$
- $c_5 = \max(c_2 - c_1, c_4 - c_3)$
  $\mathbf{c_5 = 16}$

Optimal location for the new facility is on the line connecting these two points:

$$X_1^*(x_1^*, y_1^*) = 0.5(c_1 - c_3, c_1 + c_3 + c_5) = \tfrac{1}{2}(6, 10) = \mathbf{(3, 5)}$$
$$Y_2^*(x_2^*, y_2^*) = 0.5(c_2 - c_4, c_2 + c_4 - c_5) = \tfrac{1}{2}(12, 4) = \mathbf{(6, 2)}$$

Single facility minimax location problem
Example



- Max distance equals $c_5 /2 = 16/2 =$ **8**

- The point (3,5) is 8 distance units away from $\mathbf{P_1}$,$\mathbf{P_3}$, $\mathbf{P_4}$ and $\mathbf{P_8}$, the point (6,2) is 8 distance units away from $\mathbf{P_1}$,$\mathbf{P_5}$ and $\mathbf{P_8}$ and the remaining points on the line segment are 8 distance units away from $\mathbf{P_1}$ and $\mathbf{P_8}$

Next lecture

- Facility location II.
  - Location allocation model
  - Plant location model
  - Network location problems