Special layout models

Chapter 7 (Warehouse Operations)

Chapter 10 (Facility Planning Models)

Machine layout model
Storage layout planning
Warehouse layout model

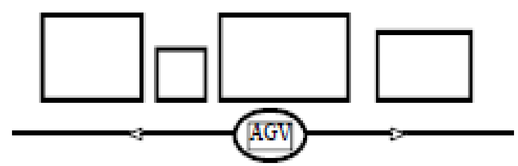
Machine Layout Models

- Objective:
  - To arrange machines on the shop floor in such a way so that the total cost is minimal.
- So far, the layout models were aggregate in nature
- Machine layout models address additional issues:
  - The interface points for incoming and outgoing parts for individual machines are usually at fixed locations (pick-up and delivery locations) relative to the entire work envelop of the machine
  - Minimum space between machines must be provided to accommodate access to machines for maintenance and service, and allow enough space for material handling devices and in-process storage areas

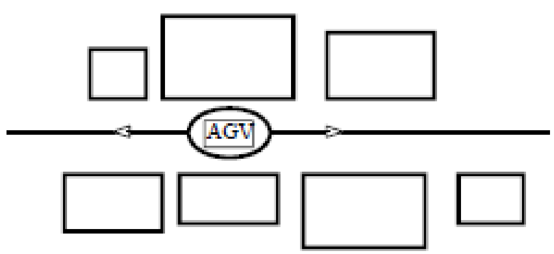
Machine Layout Models

Basic layout patterns

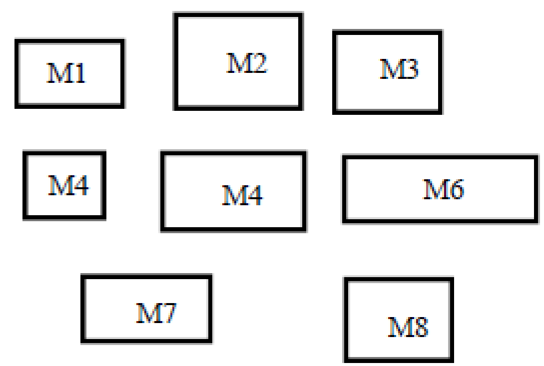
Single-Row Machine Layout



Double-Row Machine Layout



Multi-row Layout



Single row machine layout model

Let

x_i^P = location of pick-up point of machine i, $i = 1, 2, \ldots n$

x_i^D = location of delivery point of machine i

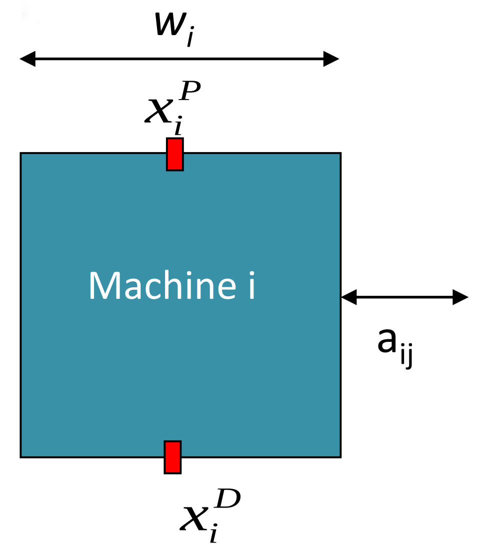
w_i = width of machine i oriented parallel to the aisle

a_{ij} = minimum clearance between machines i and j

c_{ij} = cost per trip per unit distance from machine i to j

f_{ij}^L = total loaded material handling trips from machine i to machine j

f_{ij}^E = total empty (deadhead) material handling trips from machine i to machine j



Single row machine layout model

The single-row model formulation is as follows:

$$\text{Minimize TC} = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}(f_{ij}^{L} d_{ij}^{L} + f_{ij}^{E} d_{ij}^{E})$$

The objective function gives the total cost of material handling where

$d_{ij}^{L} = |x_i^P - x_j^D|$, loaded travel distance

$d_{ij}^{E} = |x_i^D - x_j^P|$, empty (deadhead) travel distance

These constraints ensure the required distance between workstations:

$$|x_i^P - x_j^D| \geq \frac{1}{2}(w_i + w_j) + a_{ij},\ i = 1, 2, \ldots, n-1;$$

$j = i + 1, \ldots, n$ for loaded trips, and

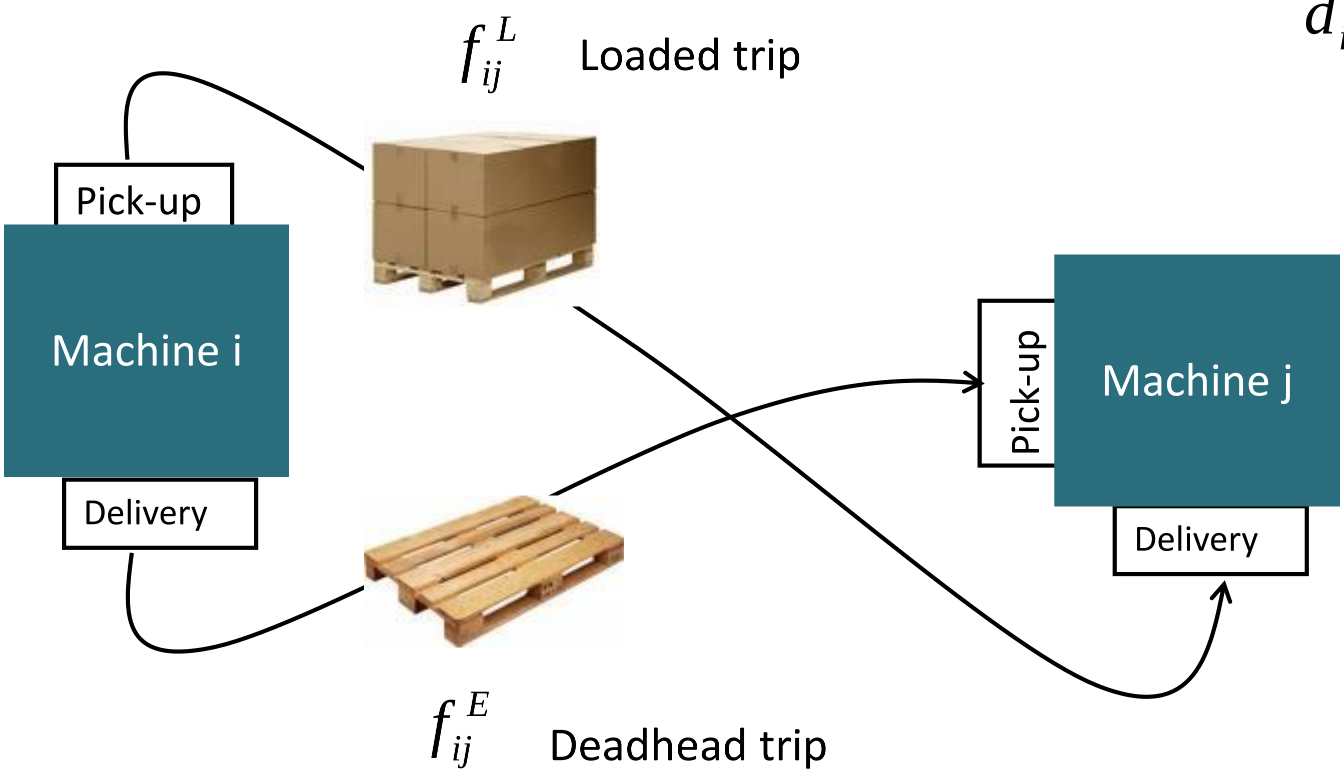
$$|x_i^D - x_j^P| \geq \frac{1}{2}(w_i + w_j) + a_{ij},\ i = 1, 2, \ldots, n-1;$$

$j = i + 1, \ldots, n$ for deahead trips.

Single row machine layout model

$$f_{ij}^{L} \neq f_{ij}^{E}$$

$$d_{ij}^{L} \neq d_{ij}^{E}$$

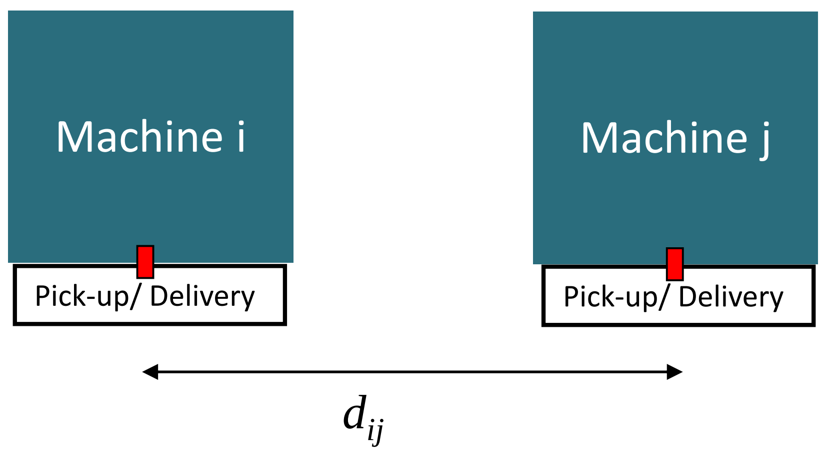


Simplified single row machine layout model

Assumptions

- We will assume that pick up and delivery point for each machine is located at the midpoint along the edge of the machine work area parallel to the aisle. $x_{ij}^{P} = x_{ij}^{D}$
  - Loaded and deadhead distances are the same

$$d_{ij} = d_{ij}^{L} = d_{ij}^{E}$$

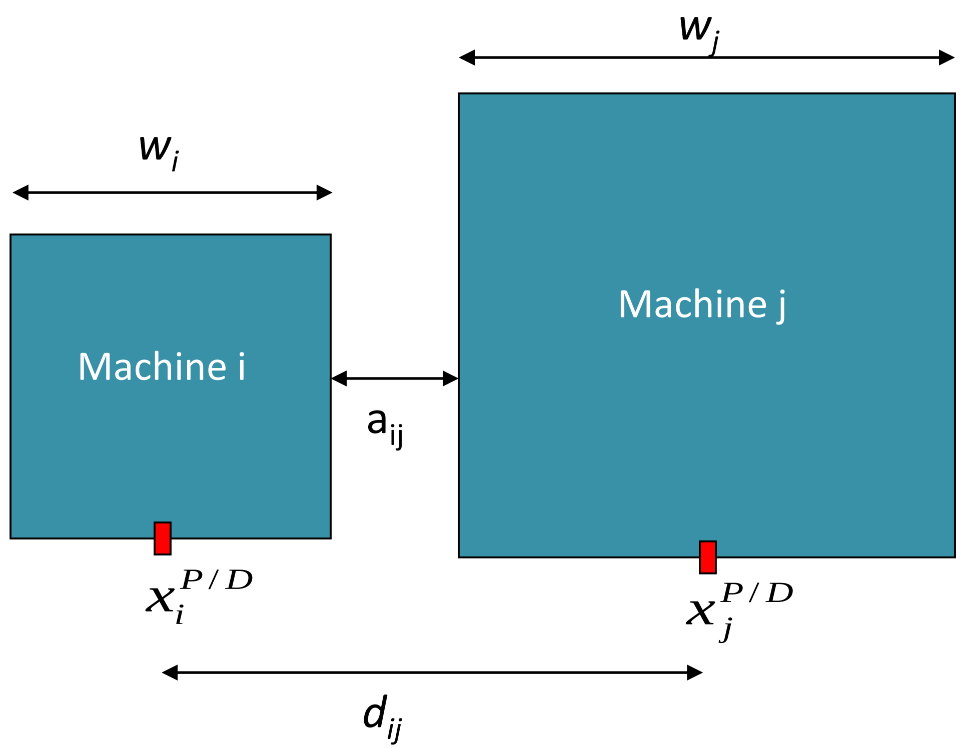


- We further assume that loaded and deadhead trips are the same

$$f_{ij} = f_{ij}^{L} + f_{ij}^{E}$$

Simplified single row machine layout model

Minimum distance



The minimum distance between Machine 1 and Machine 2:

$$\min d_{ij} = \frac{w_i + w_j}{2} + a_{ij}$$

Simplified single row machine layout model

Solution

- Determine the first 2 machines (i*and j*) to enter the layout by computing ***max*** $c_{ij}{*}f_{ij}$
- Place ***i****and ***j**** adjacent to each other. There is no difference between the placement order i*- j* or j*-i*.
- Place the next machine ***k**** in the layout. Decide whether k* will be located to the left side or to the right side of the set of machines already in the layout, based on ***Relative Placement Cost (RPC)***.

from the left **k* - i* - j*** from the right **i* - j* - k***

$$RPC = \min_{k \in U} \left\{ \sum_{i \in A} c_{ki} f_{ki} d_{ki}, \sum_{j \in A} c_{jk} f_{jk} d_{jk}, \right\}$$

- where A…set of all located machines
  U…set of all machines not yet located
- Continue until all machines are located.

Simplified single row machine layout model

Example

- Four machines should be located in a department along an aisle. The dimensions of the machines and the material handling trips between them are given below. Consider $c_{ij} = 1$ and $a_{ij} = 1$

➢ Decide about the placement order.

➢ Calculate the total cost for the optimal placement.

Number of trips

	1	2	3	4
1	-	10	5	6
2	8	-	3	8
3	7	9	-	4
4	5	11	13	-

Machine dimensions

Machine	1	2	3	4
Dimensions	2x2	3x3	4x4	5x5

Simplified single row machine layout model
Example

- Transform the From-To chart into Flow-Between chart.

Number of trips

	1	2	3	4
1	-	10	5	6
2	8	-	3	8
3	7	9	-	4
4	5	11	13	-

Flow- between matrix

	1	2	3	4
1	-	18	12	11
2		-	12	19
3			-	17
4				-

Number of trips

- Determine the first 2 machines (i*and j*) to enter the layout by computing ***max*** $\boldsymbol{c_{ij}{*}f_{ij}}$

$$max\ c_{ij}{*}f_{ij} = 19$$

- First two machines to enter the layout will be 2 and 4
- A={2,4} and U={1,3}

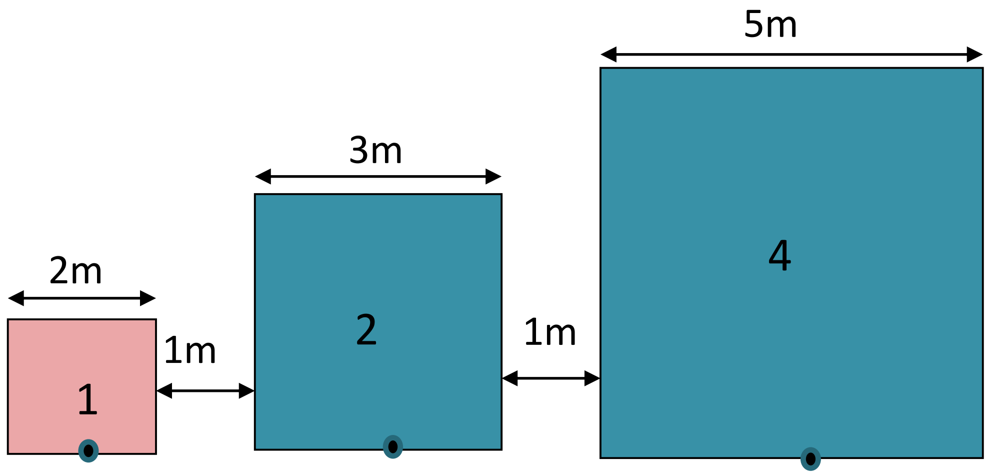
Simplified single row machine layout model

Example

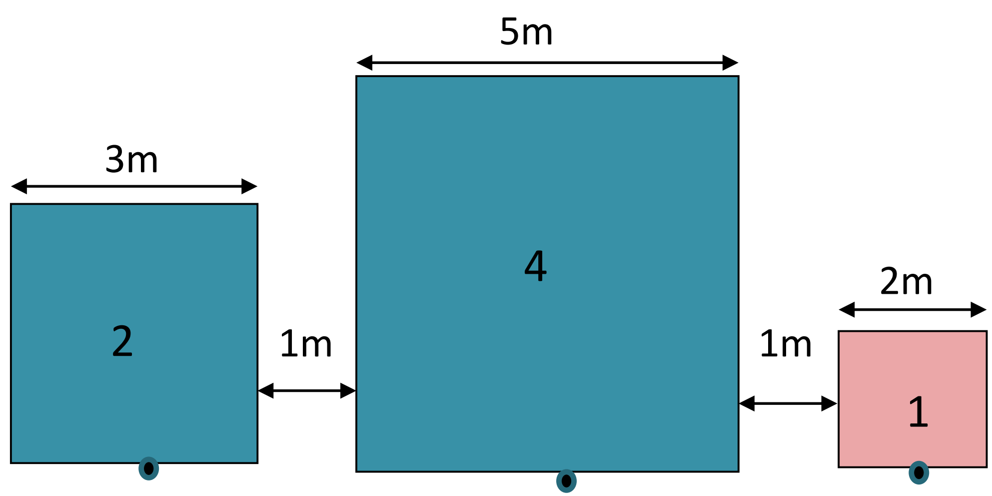
- Evaluate the possible placements of the machines 1 and 3:
- Options:
  - Machine 1:
    - From the left: 1-**2-4**
    - From the right: **2-4**-1
  - Machine 3:
    - From the left: **3-2-4**
    - From the right: **2-4-3**

Flow-between matrix

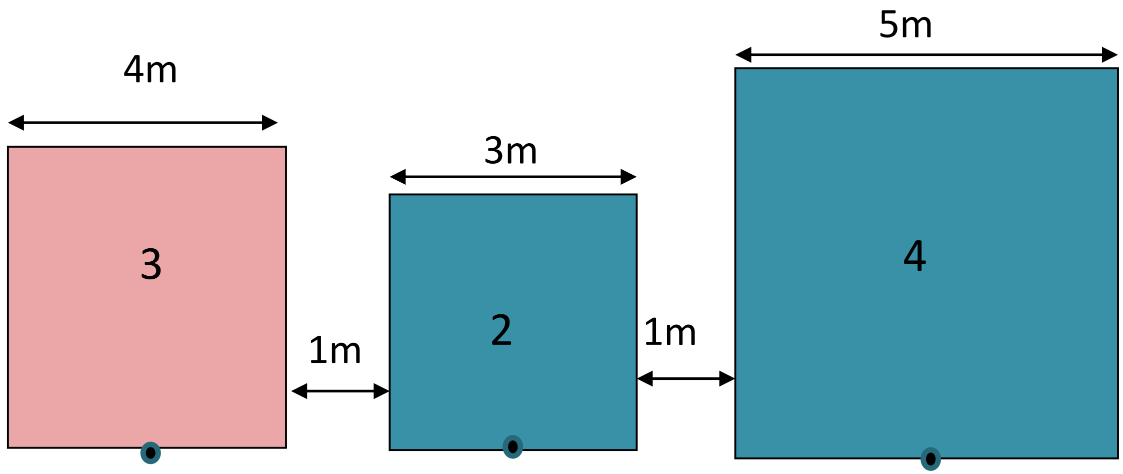
	1	**2**	**3**	**4**
1	-	18	12	11
2		-	12	19
3			-	17
4				-



- **1-2-4**
  - RPC $= f_{12}d_{12} + f_{14}d_{14} = 18*3.5 + 11*8.5 = 156.5$



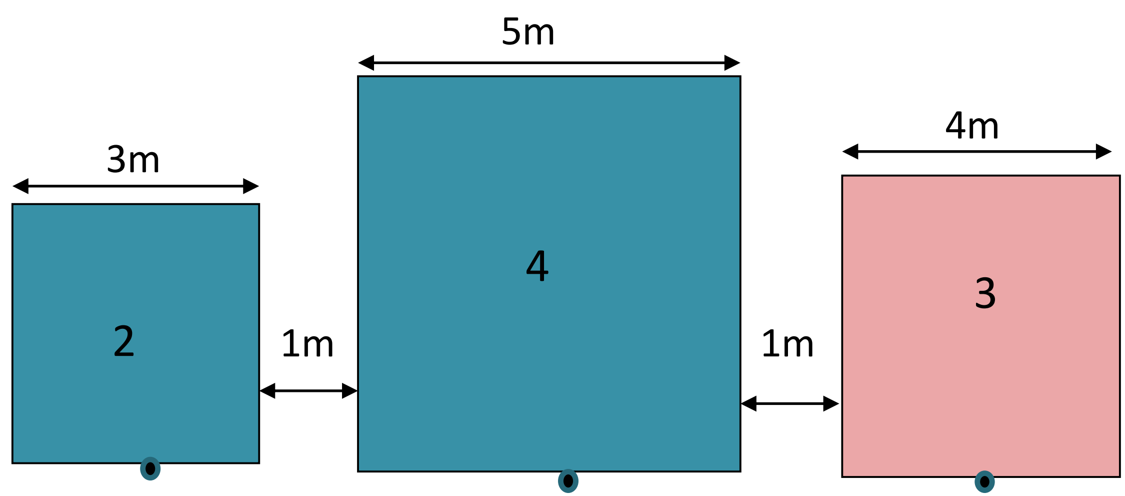
- **2-4-1**
  - RPC $= f_{21}d_{21} + f_{41}d_{41} = 18*9.5 + 11*4.5 = 220.5$



Flow-between matrix

	1	**2**	**3**	**4**
1	-	18	12	11
2		-	12	19
3			-	17
4				-

- **3-2-4**
  - RPC $= f_{32}d_{32} + f_{34}d_{34} = 12*4.5 + 17*9.5 = 215.5$



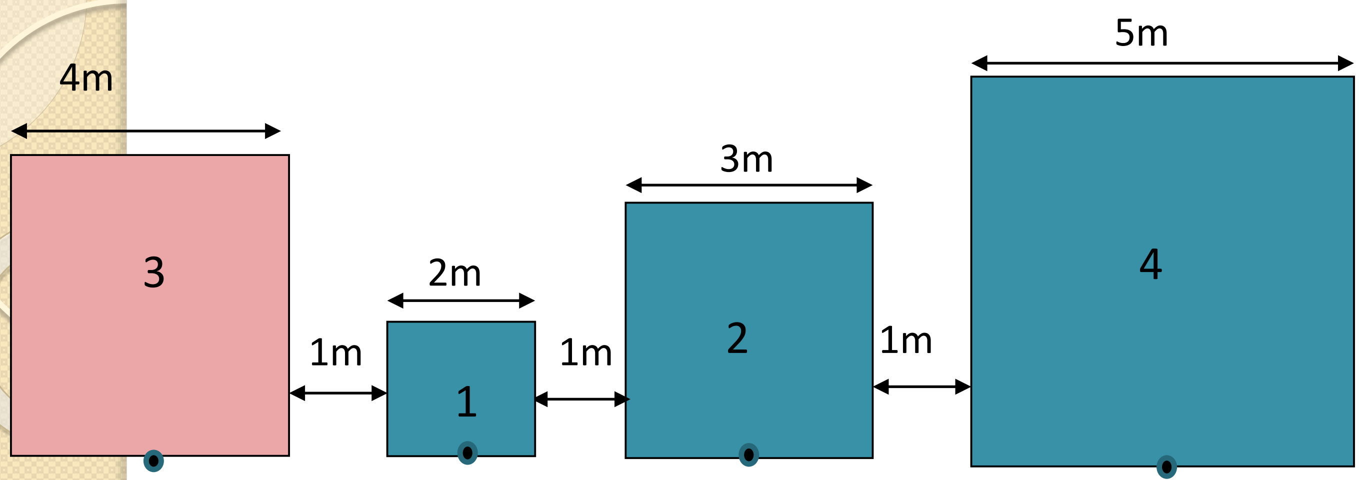
- **2-4-3**
  - RPC $= f_{23}d_{23} + f_{43}d_{43} = 12*10.5 + 17*5.5 = 219.5$

Simplified single row machine layout model
Example

- **1-2-4** ➔RPC = 156.5
- **2-4-1** ➔RPC = 220.5
- **3-2-4** ➔RPC = 215.5
- **2-4-3**➔ RPC = 219.5

- The min RPC is for order **1-2-4**

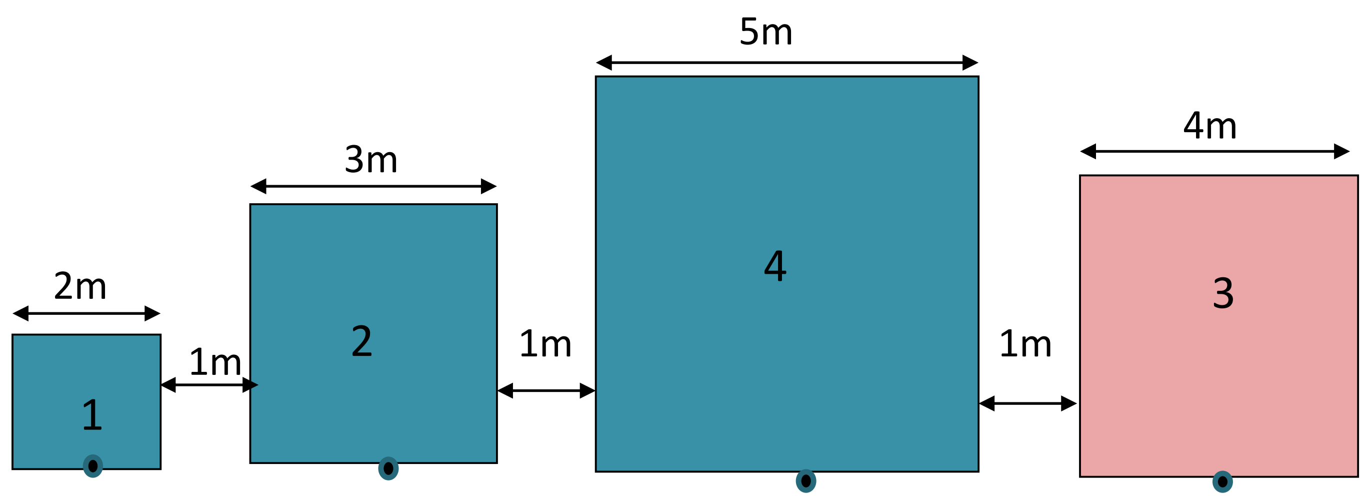
- Evaluate the possible placements of the machine 3:
  - Options:
    - From the left: **3-1-2-4**
    - From the right: **1-2-4-3**



Flow-between matrix

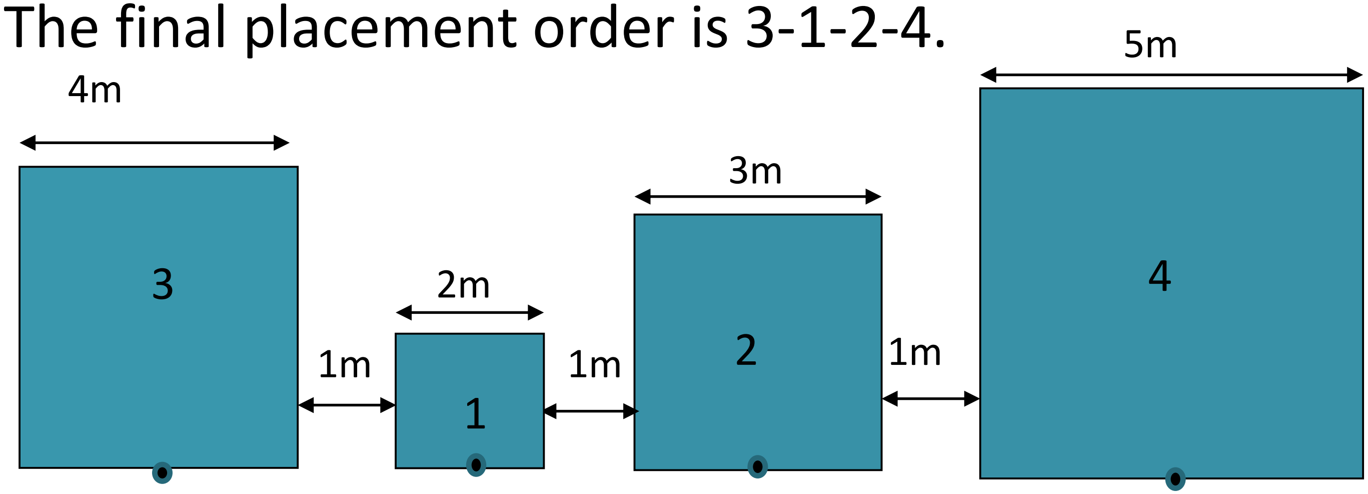
	1	2	3	4
1	-	18	12	11
2		-	12	19
3			-	17
4				-

- **3-1-2-4**
  - RPC = $f_{31}d_{31}+f_{32}d_{32} + f_{34}d_{34}$ = <u>350.5</u>



- **1-2-4-3**
  - RPC =$f_{13}d_{13}+f_{23}d_{23} + f_{43}d_{43}$ = 430

- The final placement order is 3-1-2-4.



Flow-between matrix

	1	2	3	4
1	-	18	12	11
2		-	12	19
3			-	17
4				-

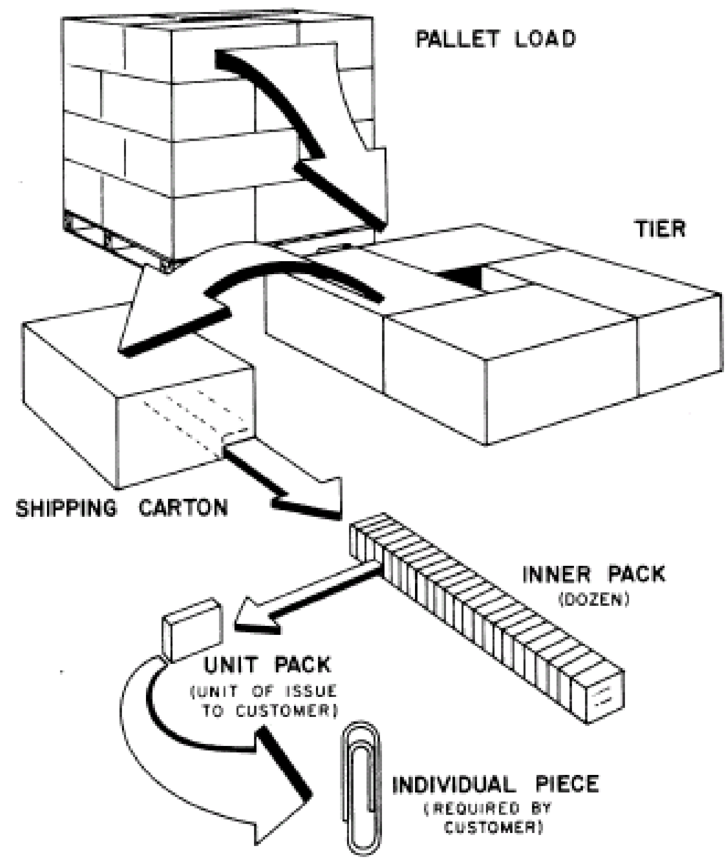
- Total cost calculation:
  - TC=18*3.5+12*4+11*8.5+12*7.5+ +19*5+17*12.5 = <u>602</u>

Storage operations

- **Storage** – the physical containment of finished goods, raw materials, supplies or in process material
- Objectives:
  - Maximize space utilization
  - Maximize equipment utilization
  - Maximize labor utilization
  - Maximize accessibility of all materials
  - Maximize protection of all materials
- Accounts for around **15%** of warehouse operating expenses
  - In general, the smaller the handling unit, the larger the handling costs

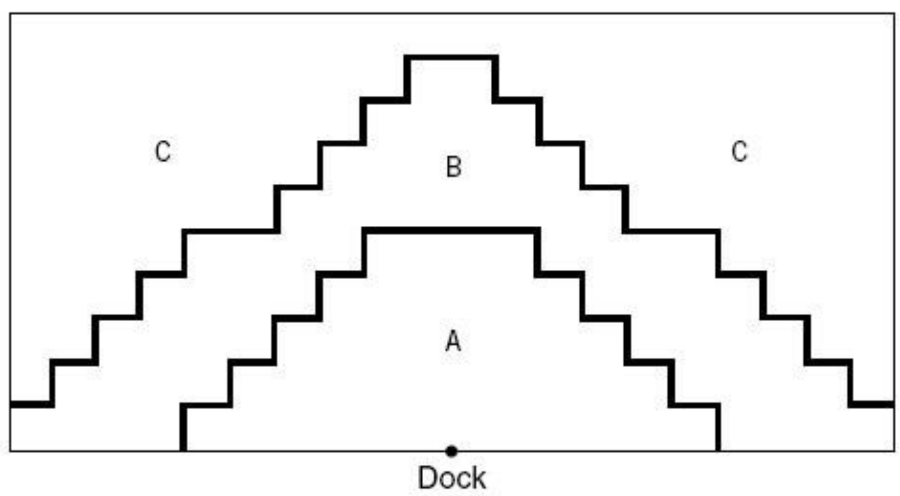
Storage operations

- **Stock Keeping Unit (SKU)** – The smallest physical unit of a product tracked by an organization



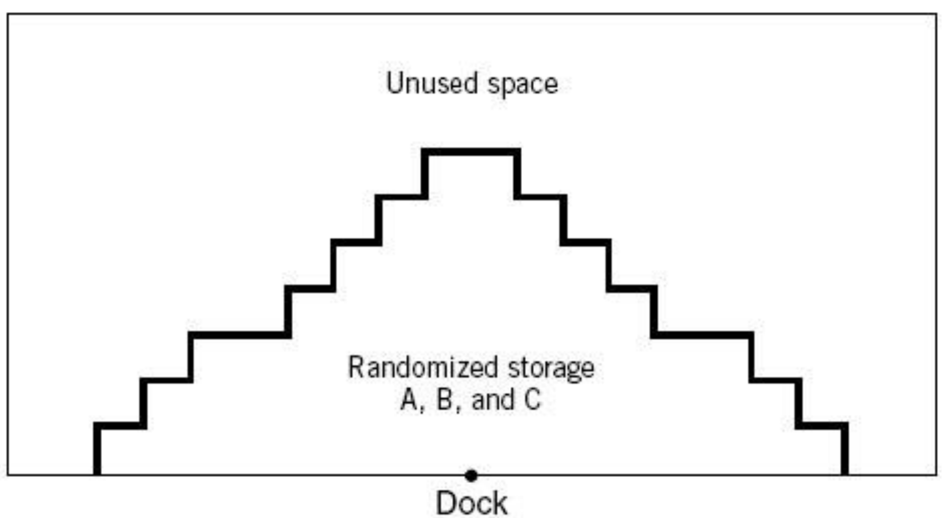
Storage Layout Methods

- ***Dedicated Storage*** (fixed-location storage)
  - Each SKU is assigned to a specific storage location or set of locations.
  - Storage locations can be arbitrarily determined, such as part number sequence, or they can be determined based on the SKU's activity level (number of storages/retrievals per unit time) and inventory level.
  - Number of storage locations is the sum of the maximum inventory level of each SKU.
  - Space requirement – to store the maximum amount ever on hand
  - Advantage: lower handling costs
    - but requires more information, careful estimates and more management



Storage Layout Methods

- ***Randomized Storage*** (random-location storage)
  - An individual stock keeping unit (SKU) can be stored in <u>any available storage location</u>.
  - Each unit of a particular product is equally likely to be retrieved when a retrieval operation is performed. Likewise each empty storage slot is equally likely to be selected for storage when a storage operation is performed.
  - Retrievals are first-in first-out (FIFO)
  - The quantity of items on hand is <u>the average amount of each SKU</u>
  - Storage requirement not known, but upper bound can be computed
  - Advantage: <u>lower space costs</u>

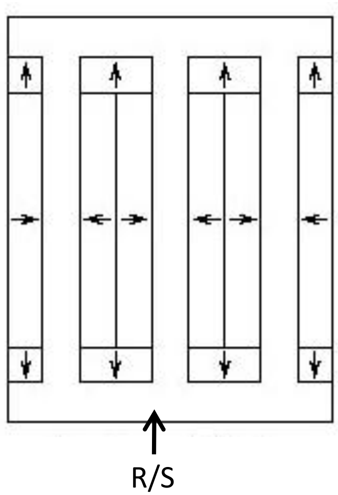


Storage layout planning

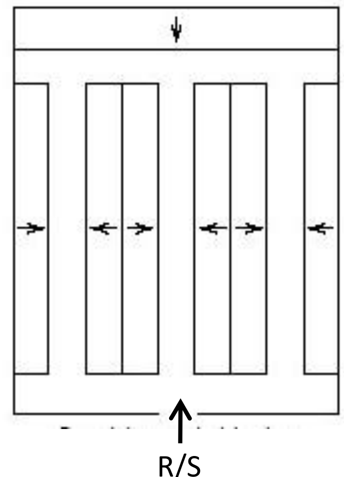
- Principles of efficient storage planning
  - Similarity
    - Receiving, shipping and storing together
  - Size
    - Variety of storage location sizes
  - Characteristics
    - Perishable, crushable, hazardous items, etc.
  - Space utilization
    - Space conservation
    - Materials accessibility
  - Popularity
    - Popular items close, in deep storage areas
    - Receiving/shipping ratio

Storage layout planning

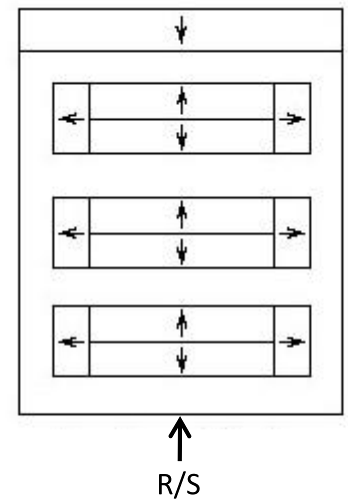
Principles – space utilization (accessibility)



Aisles should not be placed along walls without doors

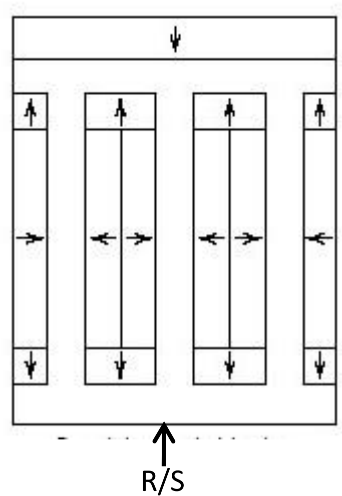


Each storage face should have an aisle access



Majority of items should be stored along the long axis of the area

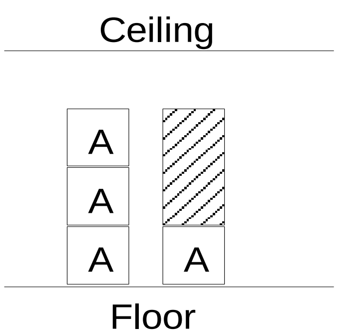
Proper layout



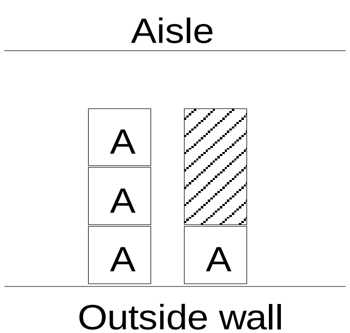
Storage layout planning

Principles – space utilization

- Honeycombing
  - Wasted space that results because a partial row or stack cannot be utilized because adding materials would result in blocked storage.

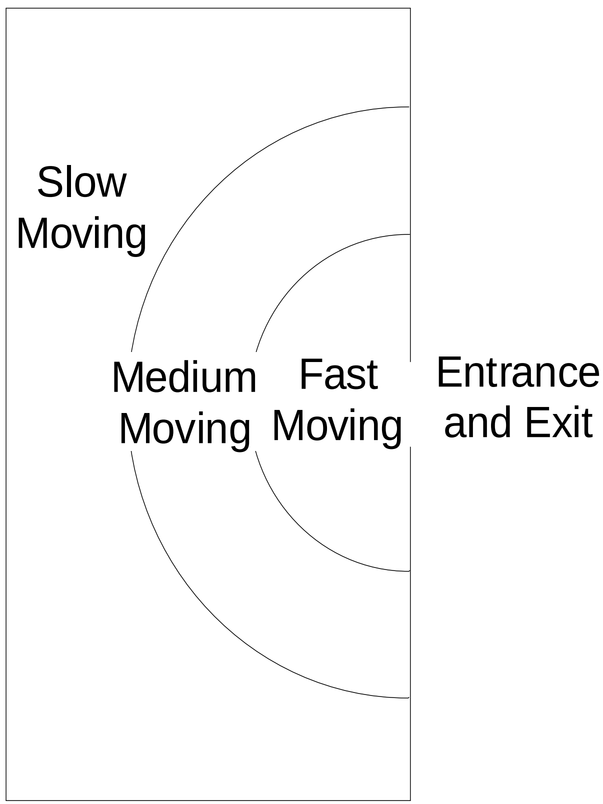


Vertical honeycombing



Horizontal honeycombing

Storage layout planning
Principles - popularity

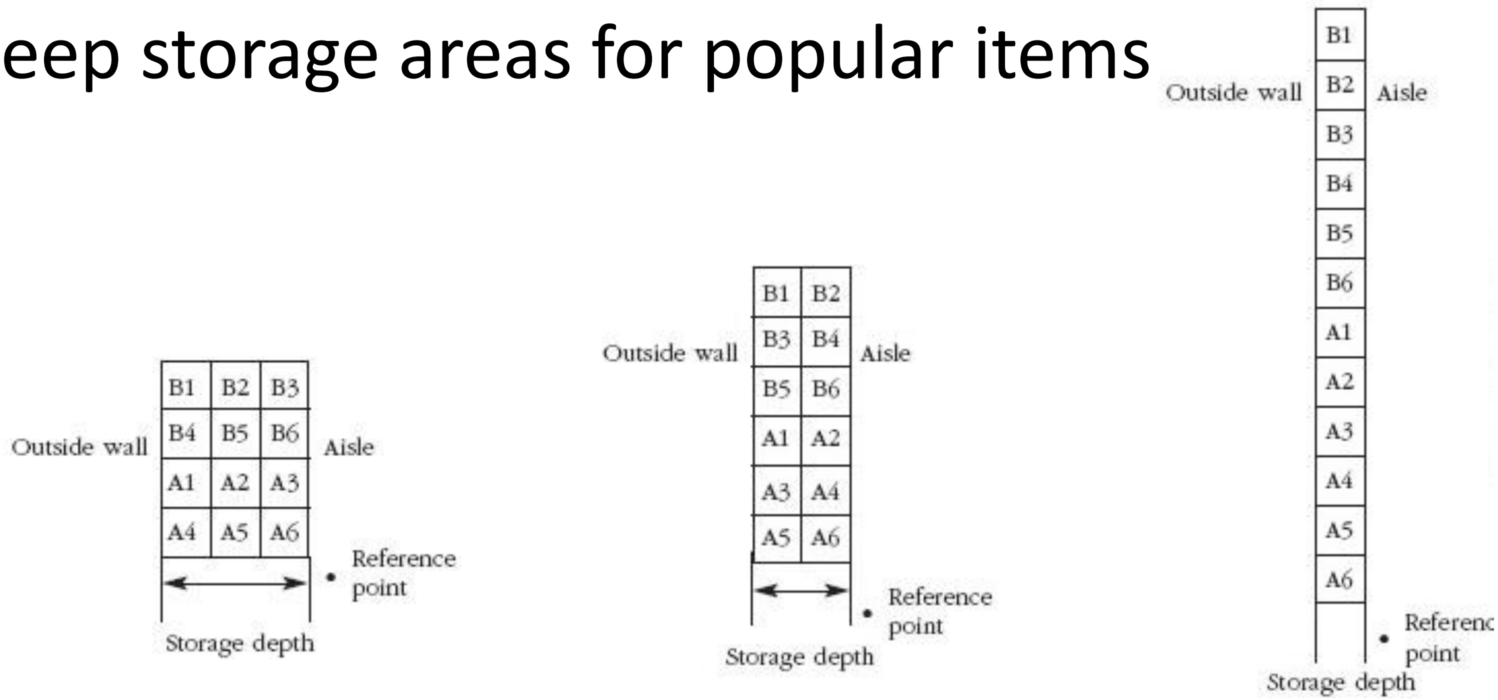


- Store the most popular items in a way that *minimizes the travel distance*
- Pareto law!
  - 85% of the turnover will be a result of 15% of the materials stored

Storage layout planning

Principles - popularity

- Deep storage areas for popular items



The impact of storage depth on travel distances:

Distances from reference point	Storage Depth 3 units	2 units	1 unit
Distance to A6	2	2	2
Distance to A1	5	5	7
Distance to B6	4	5	8
Distance to B1	7	8	13
Average travel distance	4.5	5	7.5

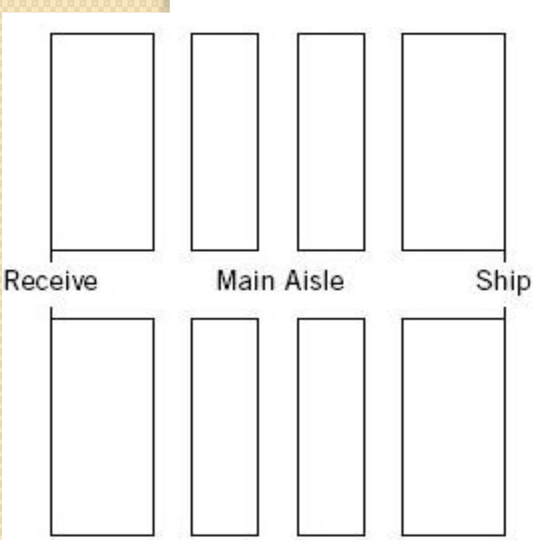
Storage layout planning

Principles - popularity

- Position based on the **Receiving/Shipping Ratio**

Example

➢ Determine the positions for the products A-H along the main aisle given the layout below and the following information:



Product	Quantity per Receipt	Trips to Receive	Average Customer Order Size	Trips to Ship
A	40 pallets	40	1.0 pallet	40
B	100 pallets	100	0.4 pallets	250
C	800 cartons	200	2.0 cartons	400
D	30 pallets	30	0.7 pallets	43
E	10 pallets	10	0.1 pallets	100
F	200 cartons	67	3.0 cartons	67
G	1000 cartons	250	8.0 cartons	125
H	1000 cartons	250	4.0 cartons	250

- Calculate Receiving/Shipping Ratio for each item

Product	Quantity per Receipt	Trips to Receive	Average Customer Order Size	Trips to Ship	Receiving/Shipping Ratio
A	40 pallets	40	1.0 pallet	40	1.0
B	100 pallets	100	0.4 pallets	250	0.4
C	800 cartons	200	2.0 cartons	400	0.5
D	30 pallets	30	0.7 pallets	43	0.7
E	10 pallets	10	0.1 pallets	100	0.1
F	200 cartons	67	3.0 cartons	67	1.0
G	1000 cartons	250	8.0 cartons	125	2.0
H	1000 cartons	250	4.0 cartons	250	1.0

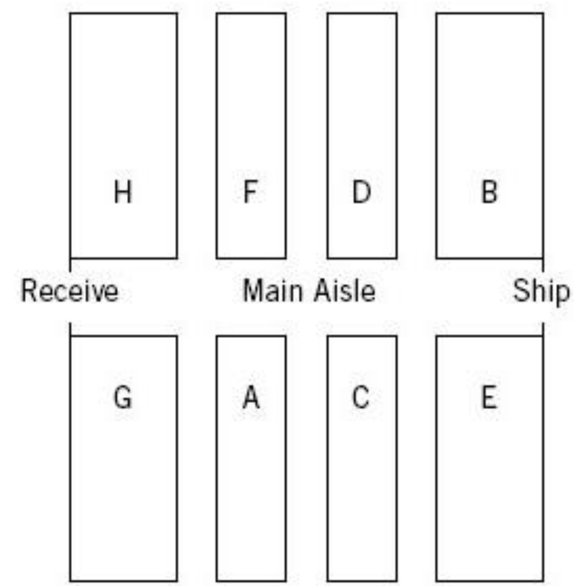
- Rules of positioning the items in the warehouse:
  - ratio < 1.00 => closer to shipping
  - ratio > 1.00 => closer to receiving
- Position the items in order of importance of being close to shipping or receiving

E – B – C – D – A – F – H – G

Closer to shipping

Closer to receiving

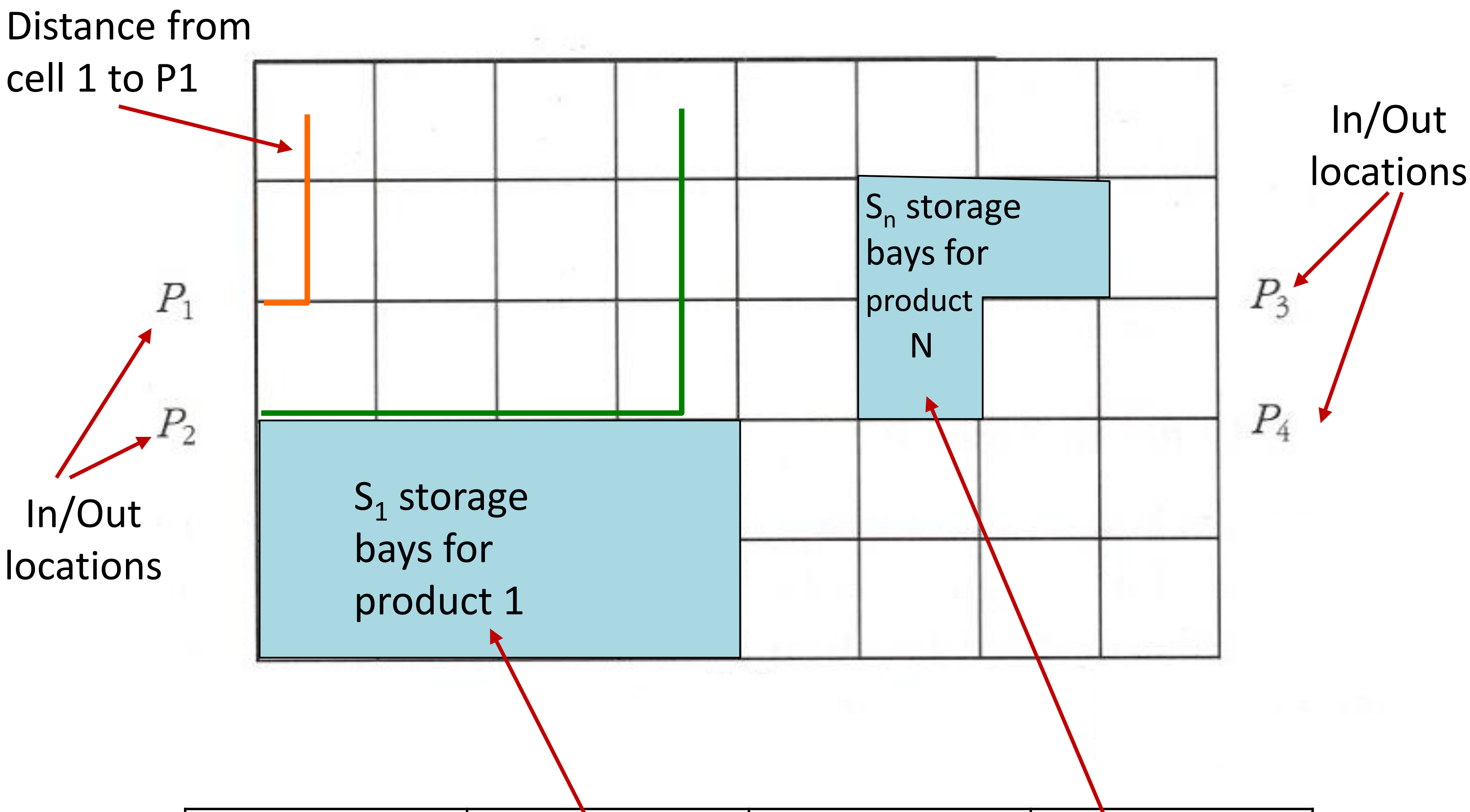
Travel distance the same



Warehouse layout model

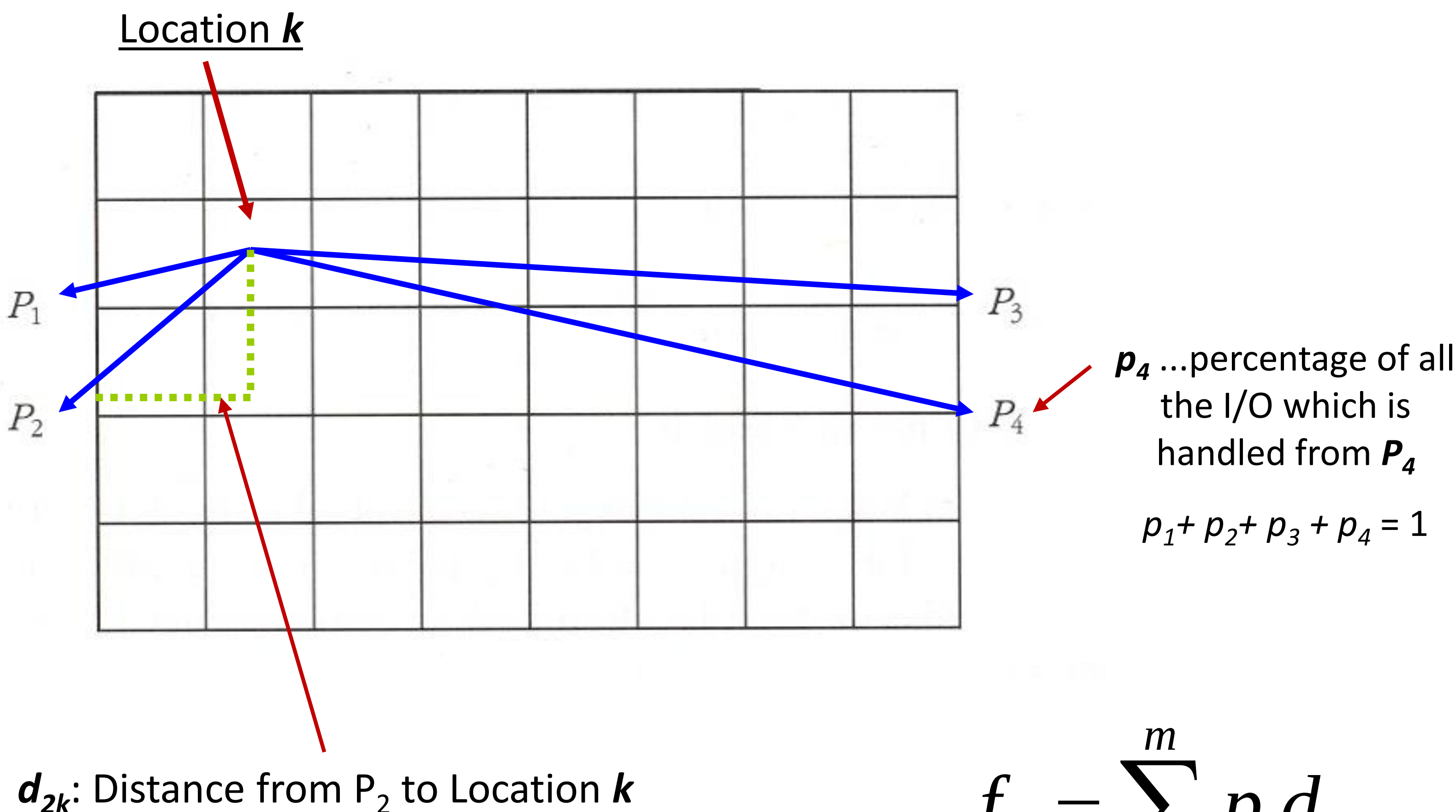
- Quantitative model for determination of the location of products for storage and warehouse
- We assume
  - Dedicated storage layout
  - Rectilinear distances

Warehouse layout model



Products	1	…	N
Required number of storage bays	S_1	…	S_n

Warehouse layout model



$$f_k = \sum_{i=1}^{m} p_i d_{ik}$$

- The objective is to minimize f_k which is the expected distance traveled between storage location k and the docks

Warehouse layout model

- **Procedure for warehouse design which minimizes the distance**:

1. Number the products according to their T_j/S_j

$$\frac{T_1}{S_1} \geq \frac{T_2}{S_2} \geq \cdots \geq \frac{T_n}{S_n}$$

where T_j…… number of in/out trips for product *j*

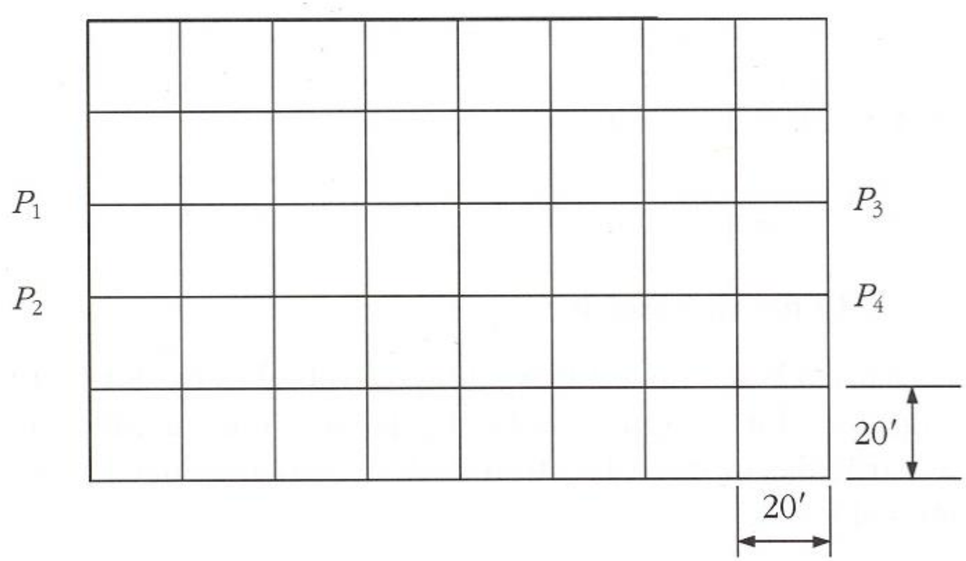
S_j …..number of storage locations required for product *j*

1. Compute the f_k values for all storage locations
2. Assign product 1 to the S_1 storage locations having the lowest f_k values; assign product 2 to the S_2 storage locations having the next lowest f_k values; and so on….

Warehouse layout model
Example

- Warehouse given below has four docks. Docks P1 and P2 are for truck delivery (60% of the all the movement, with each dock equally likely to be used) and docks P3 and P4 are for rail delivery (remaining 40% is equally divided between P3 and P4)
- There are 3 products A (3600 ft^2 with 750 loads per month), B (6400 ft^2 with 900 loads per month) and C (4000ft^2 with 800 loads per month) which should be stored in the warehouse.

➢ Design a warehouse layout



Warehouse layout model
Example

1. Number the products according to their T_j/S_j

- Number of storage bays required for each product:

S_A = 3600/400 = 9
S_B = 6400/400 = 16
S_C = 4000/400 = 10

- Calculate T_j/S_j for each product:

$$\frac{T_A}{S_A} = \frac{750}{9} = 83.33; \quad \frac{T_B}{S_B} = \frac{900}{16} = 56.25; \quad \frac{T_C}{S_C} = \frac{800}{10} = 80$$

- Rank the products:

$$\frac{T_A}{S_A} \geq \frac{T_C}{S_C} \geq \frac{T_B}{S_B} \Rightarrow$$

1. A
2. C
3. B

Warehouse layout model
Example

2. Compute the $\boldsymbol{f_k}$ values for all storage locations

$$f_k = \sum_{i=1}^{m} p_i d_{ik}$$

$$f_1 = 0.3*40 + 0.3*60 + 0.2*180 + 0.2*200 = 106$$

$$f_{29} = 0.3*120 + 0.3*100 + 0.2*100 + 0.2*80 = 102$$

$w_1 = 0.3\ P_1$

$w_2 = 0.3\ P_2$

106_{1}	110_{2}	114_{3}	118_{4}	122_{5}	126_{6}	130_{7}	134_{8}
86_{9}	90_{10}	94_{11}	98_{12}	102_{13}	106_{14}	110_{15}	114_{16}
76_{17}	80_{18}	84_{19}	88_{20}	92_{21}	96_{22}	100_{23}	104_{24}
86_{25}	90_{26}	94_{27}	98_{28}	102_{29}	106_{30}	110_{31}	114_{32}
106_{33}	110_{34}	114_{35}	118_{36}	122_{37}	126_{38}	130_{39}	134_{40}

$P_3\ w_3 = 0.2$

$P_4\ w_4 = 0.2$

Warehouse layout model
Example

3. Assign product 1 to the S_1 storage locations having the lowest f_k values; assign product 2 to the S_2 storage locations having the next lowest f_k values; and so on….

1. A $S_A = 9$
2. C $S_C = 10$
3. B $S_B = 16$

Cell#	Fj	Product	Cell#	Fj	Product	Cell#	Fj	Product	Cell#	Fj	Product
17	76	A	11	94	C	14	106	B	38	126	
18	80	A	27	94	C	30	106	B	7	130	
19	84	A	22	96	C	33	106	B	39	130	
9	86	A	12	98	C	2	110	B	8	134	
25	86	A	28	98	C	15	110	B	40	134	
20	88	A	23	100	C	31	110	B			
10	90	A	13	102	C	34	110	B			
26	90	A	29	102	C	3	114	B			
21	92	A	24	104	C	16	114	B			
			1	106	C	32	114	B			
						35	114	B			
						4	118	B			
						36	118	B			
						5	122	B			
						37	122	B			
						6	126	B			

The remaining storage bays are available for equipment, WC, offices, etc.

Warehouse layout model
Example

P_1 P_2

C	B	B	B	B	B		
A	A	C	C	C	B	B	B
A	A	A	A	A	C	C	C
A	A	C	C	C	B	B	B
B	B	B	B	B			

P_3 P_4

Final layout which minimizes expected distance traveled per unit time

Next lecture

Quiz #4